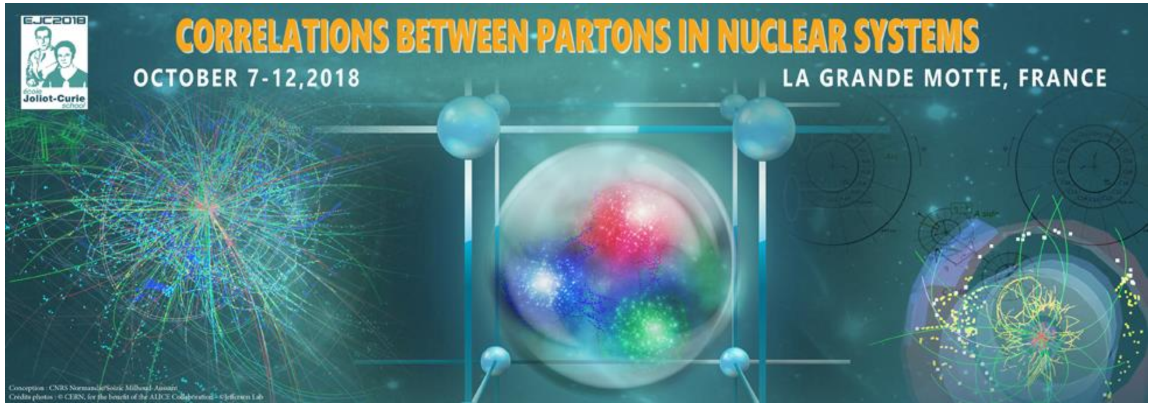


Neutron stars and nucleons: Are they so different?

Based on **[C.L., Eur. Phys. J. C78 (2018)]**
[C.L., Moutarde, Trawinski, *in preparation*]

Cédric Lorcé

CPHT ÉCOLE POLYTECHNIQUE UNIVERSITÉ PARIS-SACLAY

October 9, La Grande Motte, France

Outline

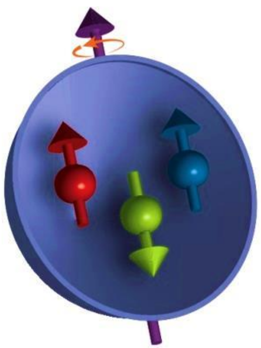
1. **Nucleon structure**
2. **Energy-momentum tensor**
3. **Mass decompositions**
4. **3D distributions in Breit frame**
5. **Comparison with neutron stars**
6. **Summary**

Origin of mass and spin?

Non-relativistic picture
dominated by constituents

Until ~ 1980

Spectroscopy



Mass

$$M_N \sim \sum_Q M_Q + E_{\text{binding}}$$

~ 102 % ~ - 2 %

Spin

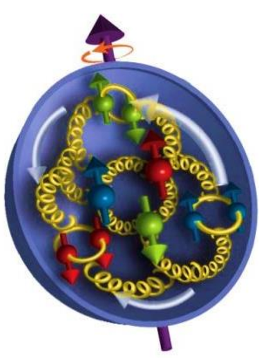
$$J_z^N \sim \sum_Q S_z^Q$$

~ 100 %

Relativistic picture
dominated by dynamics

Now

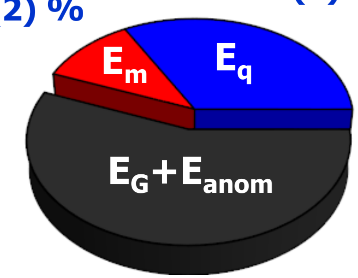
High-energy scattering



Higgs mechanism

~ 13(2) %

~ 31(2) %



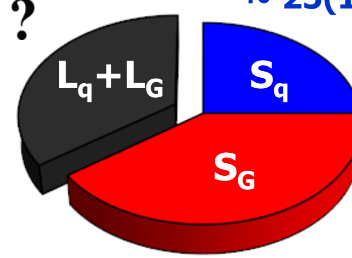
?

Gluon and quantum anomaly

Orbital angular momentum

?

~ 25(10) %



~ 40(?) %

Hot news

PREPUBLICATION COPY—SUBJECT TO FURTHER EDITORIAL CORRECTION

An Assessment of
U.S.-Based Electron-Ion Collider Science

Committee on U.S.-Based Electron-Ion Collider Science Assessment

Board on Physics and Astronomy

Division on Engineering and Physical Sciences

A Consensus Study Report of

The National Academies of
SCIENCES · ENGINEERING · MEDICINE

THE NATIONAL ACADEMIES PRESS
Washington, DC
www.nap.edu

PREPUBLICATION COPY—SUBJECT TO FURTHER EDITORIAL CORRECTION

Hearing from experts on the science that an EIC would be able to carry out, the committee finds that

Finding 1: An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

Summary extract from a report by the U.S. National Academy of Sciences on the science of U.S.-based Electron-Ion Collider (July 24, 2018)

First experimental extraction of the pressure distribution inside the proton

[Burkert, Elouadrhiri, Girod, *Nature 557* (2018)]

LETTER

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1*], L. Elouadrhiri[1] & F. X. Girod[1]

10x the pressure @ center of neutron stars!

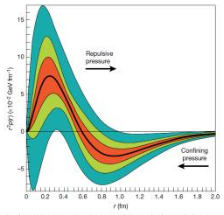


Fig. 1 | Radial pressure distribution in the proton.

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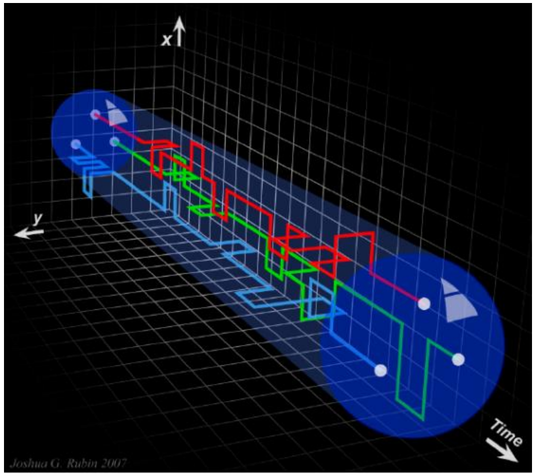
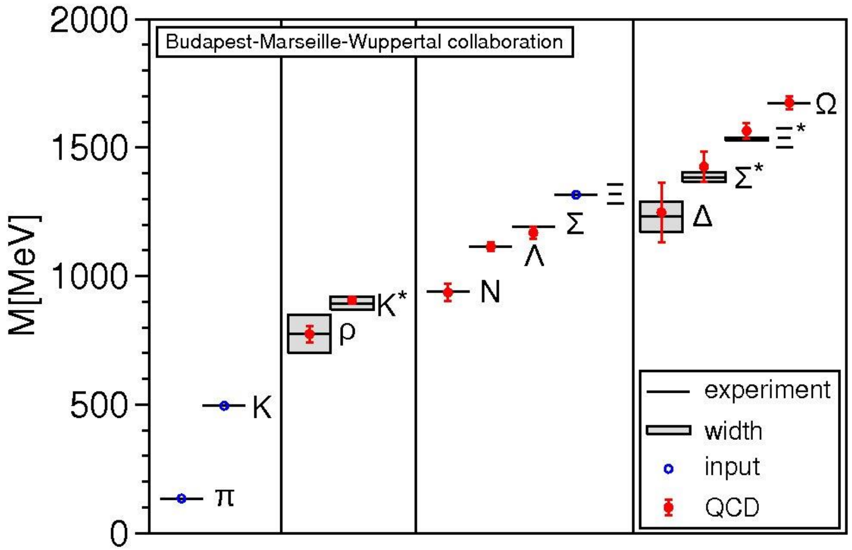
Soon brand new data from JLab 12GeV and COMPASS II!

Lattice QCD

Ab initio mass calculation based on Euclidean space-time correlators $\sim \sum_n e^{-E_n \tau}$



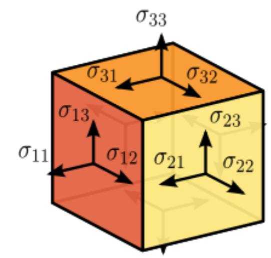


[Dürr *et al.* (2008)]

Unfortunately, little insight on where mass comes from …

Energy-momentum tensor (EMT)

Mass, **spin** and **pressure** all encoded in



Energy density **Momentum density**

$$T^{\mu\nu} = \begin{bmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{bmatrix}$$

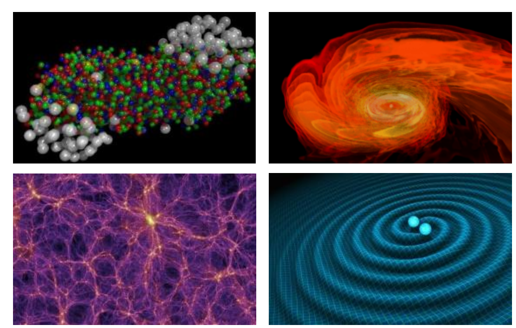
Shear stress

Normal stress **(pressure)**

Energy flux **Momentum flux**

Key concept for

- **Nucleon mechanical properties**
- **Quark-gluon plasma**
- **Relativistic hydrodynamics**
- **Stellar structure and dynamics**
- **Cosmology**
- **Gravitational waves**
- **Modified theories of gravitation**
- **...**



Gravitational form factors (GFFs)

[Kobzarev, Okun (1962)]
[Pagels (1966)]
[Ji (1996)]
[Bakker, Leader, Trueman (2004)]
[Leader, C.L. (2014)]

Matrix elements $P = \frac{p' + p}{2}, \qquad \Delta = p' - p, \qquad t = \Delta^2$

$$\langle p', s'|T^{\mu\nu}(0)|p, s\rangle =$$

$$\overline{u}(p', s') \left[\frac{P^\mu P^\nu}{M} A(t) + \frac{P^{\{\mu} i\sigma^{\nu\}\lambda}\Delta_\lambda}{4M} (A+B)(t) + \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu}\Delta^2}{M} C(t) \right.$$

$$\left. + \boxed{M g^{\mu\nu} \bar{C}(t)} + \boxed{\frac{P^{[\mu} i\sigma^{\nu]\lambda}\Delta_\lambda}{4M} D(t)} \right] u(p, s)$$

Non-conservation **Intrinsic spin**

Mass-shell conditions

$$p'^2 = p^2 = M^2 \quad \Longrightarrow \quad P^2 = M^2 - \frac{t}{4}, \qquad P \cdot \Delta = 0$$

Timelike → **« frame »**

Spacelike → **« position »**

Quantum chromodynamics (QCD)

Classical QCD energy-momentum tensor

$$T^{\mu\nu} = \overline{\psi}\gamma^\mu \tfrac{i}{2}\overleftrightarrow{D}^\nu \psi - G^{a\mu\alpha}G^{a\nu}{}_\alpha + \tfrac{1}{4}\,\eta^{\mu\nu}G^2$$

Renormalized trace of the QCD EMT

[Crewther (1972)]
[Chanowitz, Ellis (1972)]
[Nielsen (1975)]
[Adler, Collins, Duncan (1977)]
[Collins, Duncan, Joglekar (1977)]
[Nielsen (1977)]

$$T^\mu{}_\mu = \underbrace{\frac{\beta(g)}{2g}\,G^2}_{\text{Trace anomaly}} + (1+\gamma_m)\,\overline{\psi}\underset{\substack{\uparrow\\ \text{Quark mass matrix}}}{m}\psi$$

Poincaré invariance

[Kobzarev, Okun (1962)]
[Teryaev (1999)]
[Brodsky, Hwang, Ma, Schmidt (2001)]
[Leader, C.L. (2014)]
[Teryaev (2016)]
[Lowdon, Chiu, Brodsky (2017)]

$$\partial_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \sum_{i=q,g} A_i(0) = 1, \qquad \sum_{i=q,g} \bar{C}_i(t) = 0$$

$$\partial_\mu \underbrace{J^{\mu\alpha\beta}}_{x^\alpha T^{\mu\beta} - x^\beta T^{\mu\alpha} + S^{\mu\alpha\beta}} = 0 \quad \Rightarrow \quad \sum_{i=q,g} B_i(0) = 0, \qquad D_q(t) = -G^q_A(t)$$

Textbook decomposition

Forward matrix element

$$\langle P|T^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu$$

$$\langle P'|P\rangle = 2P^0\,(2\pi)^3\,\delta^{(3)}(\vec{P}'-\vec{P})$$

Trace decomposition

$$\begin{aligned} 2M^2 &= \langle P|T^\mu{}_\mu(0)|P\rangle \\ &= \underbrace{\langle P|\frac{\beta(g)}{2g}\,G^2|P\rangle}_{\sim\,89\%} + \underbrace{\langle P|(1+\gamma_m)\,\overline{\psi}m\psi|P\rangle}_{\sim\,11\%} \end{aligned}$$

[Shifman, Vainshtein, Zakharov (1978)]
[Luke, Manohar, Savage (1992)]
[Donoghue, Golowich, Holstein (1992)]
[Kharzeev (1996)]
[Bressani, Wiedner, Filippi (2005)]
[Roberts (2017)]
[Krein, Thomas, Tsushima (2017)]

- ✓ Manifestly covariant
- ✓ Compatible with Gell-Mann–Oakes–Renner formula for pion

- ✗ Depends on state normalization
- ✗ No spatial extension
- ✗ No clear relation to energy

Ji's decomposition

[Ji (1995)]

Separation of quark and gluon contributions

$$T^{\mu\nu} = \underbrace{\bar{T}^{\mu\nu}}_{\text{Traceless}} + \underbrace{\hat{T}^{\mu\nu}}_{\text{Pure trace}} \qquad \bar{T}^{\mu\nu} = \bar{T}^{\mu\nu}_q + \bar{T}^{\mu\nu}_g \qquad \hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_m + \hat{T}^{\mu\nu}_a$$

Forward matrix elements

$$\langle P|\bar{T}^{\mu\nu}_i(0)|P\rangle = 2\left(P^\mu P^\nu - \tfrac{1}{4}\,\eta^{\mu\nu} M^2\right) A_i(0)$$
$$\langle P|\hat{T}^{\mu\nu}_i(0)|P\rangle = \tfrac{1}{2}\,\eta^{\mu\nu} M^2 \left[A_i(0) + 4\bar{C}_i(0)\right]$$

$$\langle O\rangle = \frac{\langle P|\int \mathrm{d}^3r\, O(r)|P\rangle}{\langle P|P\rangle}$$

Ji's decomposition [Gao *et al.* (2015)]

$$M = \underbrace{M_q}_{\sim 31\%} + \underbrace{M_g}_{\sim 34\%} + \underbrace{M_m}_{\sim 13\%} + \underbrace{M_a}_{\sim 22\%}$$

$\mu = 2\,\mathrm{GeV}$

$$M_q = \langle \bar{T}^{00}_q\rangle|_{\vec{P}=\vec{0}} - \tfrac{3}{1+\gamma_m}\,\langle \hat{T}^{00}_m\rangle|_{\vec{P}=\vec{0}}$$
$$M_g = \langle \bar{T}^{00}_g\rangle|_{\vec{P}=\vec{0}}$$
$$M_m = \tfrac{4+\gamma_m}{1+\gamma_m}\,\langle \hat{T}^{00}_m\rangle|_{\vec{P}=\vec{0}}$$
$$M_a = \langle \hat{T}^{00}_a\rangle|_{\vec{P}=\vec{0}}$$

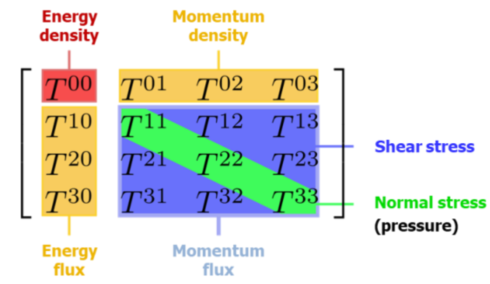
- ✔ **Proper normalization**
- ✔ **Clear relation to energy distribution**
- ! **Scale-dependent interpretation in the rest frame**
- ✘ **Pressure effects not taken into account**

New decomposition

[C.L. (2017)]

Forward matrix element

$$\langle P|T_i^{\mu\nu}(0)|P\rangle = 2P^\mu P^\nu A_i(0) + 2M^2\eta^{\mu\nu}\bar{C}_i(0)$$



Analogy with relativistic hydrodynamics

Perfect fluid element

$$\Theta_i^{\mu\nu} = (\varepsilon_i + p_i)u^\mu u^\nu - p_i\,\eta^{\mu\nu}$$

Four-velocity

$$u^\mu = P^\mu/M$$

Energy density

$$\varepsilon_i = [A_i(0) + \bar{C}_i(0)]\,\frac{M}{V}$$

Isotropic pressure

$$p_i = -\bar{C}_i(0)\,\frac{M}{V}$$

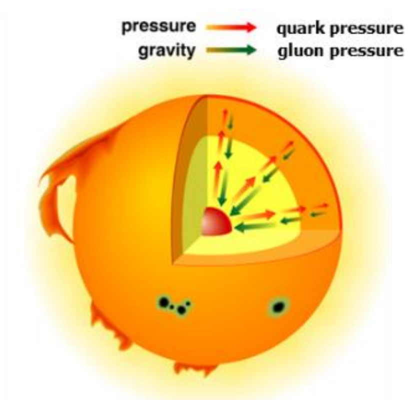
Nucleon mass decomposition $U_i = \varepsilon_i V$

$$M = \underbrace{U_q}_{\sim 44\%} + \underbrace{U_g}_{\sim 56\%}$$

$$p_q = -p_g$$

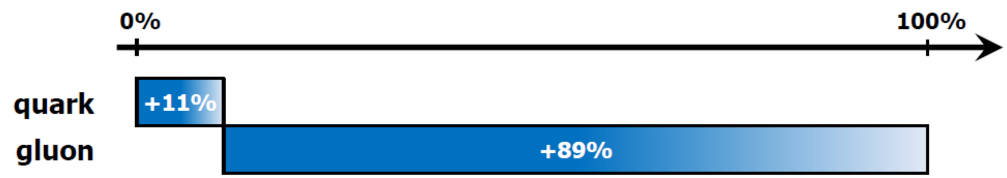
~ 11%

$\mu = 2\,\mathrm{GeV}$

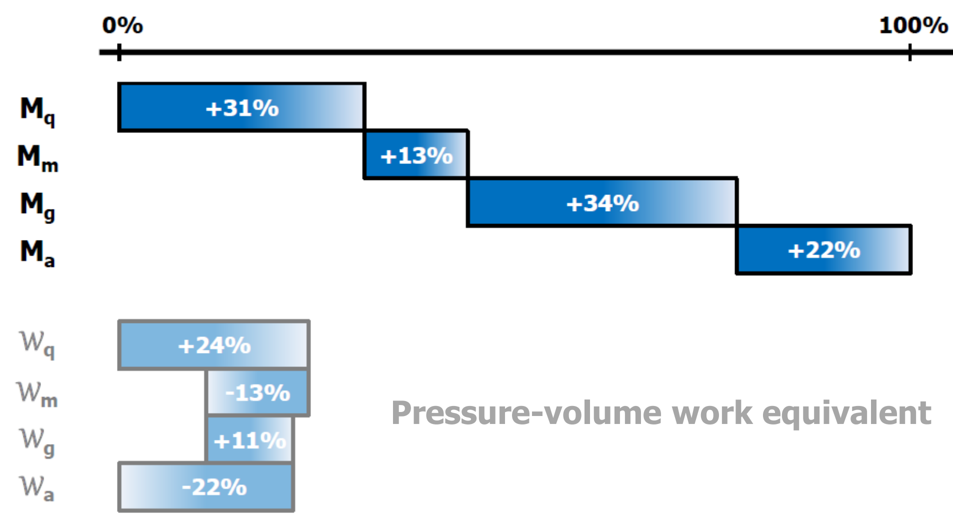


In short

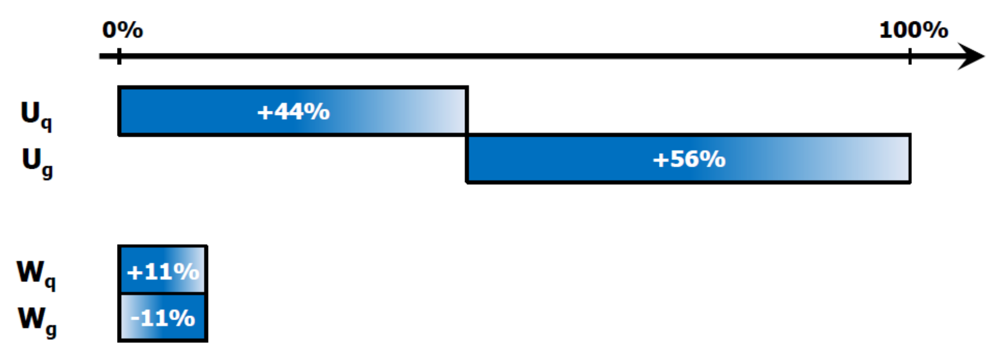
Trace decomposition



Ji's decomposition



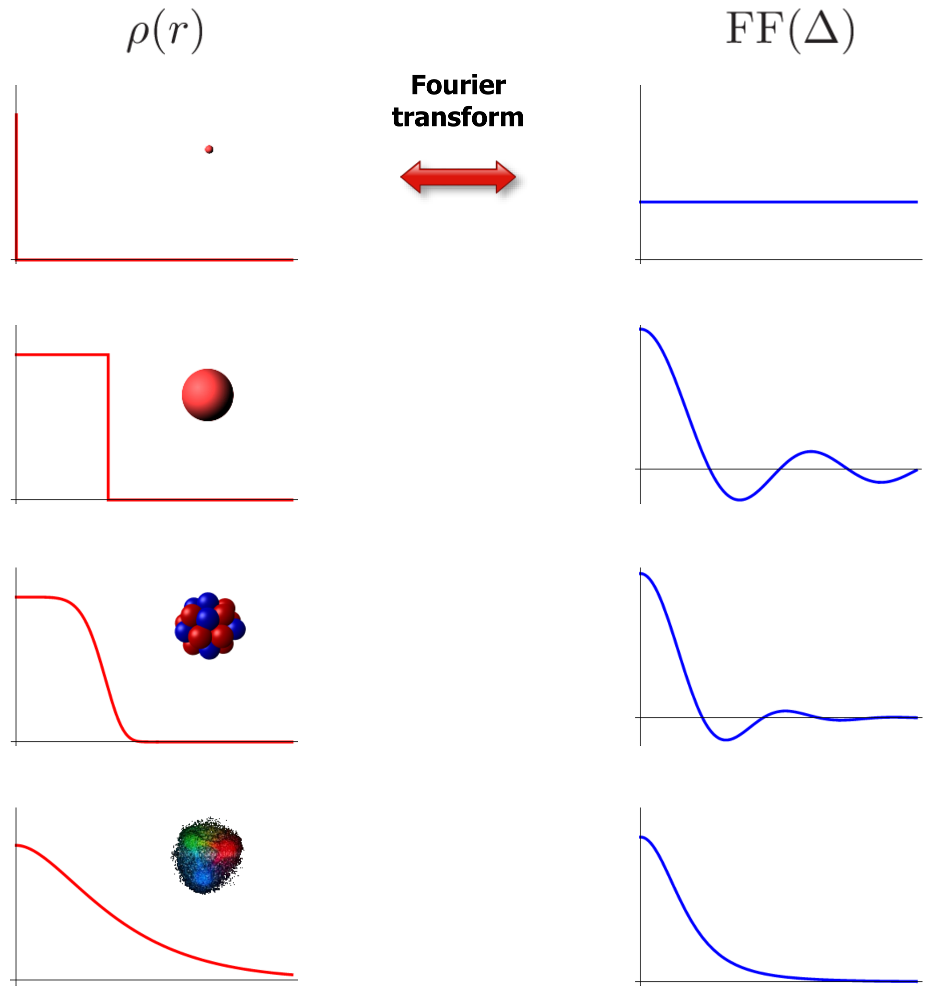
New decomposition



$\mu = 2\,\mathrm{GeV}$

Spatial information

Charge distribution | **Electric form factor**

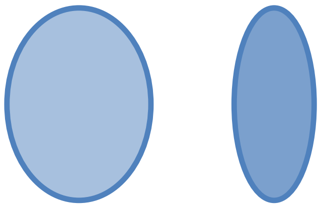


3D distribution in Breit frame

Lorentz factors

$$p^0 = \gamma M, \qquad p'^0 = \gamma' M$$

Initial ψ **Final** ψ'^*

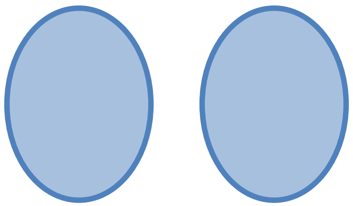


$\Delta^0 \neq 0$

Breit frame

$$\xrightarrow{\vec{p}\,'} \qquad \xleftarrow[\vec{p}]{}$$

$$\vec{P} = \vec{0} \quad \Rightarrow \quad \Delta^0 = \frac{\vec{P}\cdot\vec{\Delta}}{P^0} = 0$$



$\Delta^0 = 0$

$\rho \sim \psi^* \psi$

3D distribution

$$T_i^{\mu\nu}(\vec{r}) = \int \frac{\mathrm{d}^3\Delta}{(2\pi)^3 2P^0}\, e^{-i\vec{\Delta}\cdot\vec{r}}\, \langle \tfrac{\vec{\Delta}}{2} | T_i^{\mu\nu}(0) | -\tfrac{\vec{\Delta}}{2} \rangle$$

Anisotropic medium

[Polyakov (2003)]
[Goeke *et al.* (2007)]
[Polyakov, Schweitzer (2018)]
[C.L., Moutarde, Trawinski, *in preparation*]

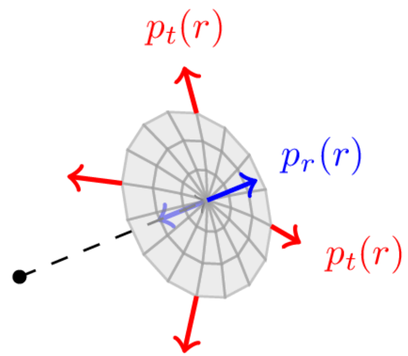
Breit frame amplitude $t = -\vec{\Delta}^2$

$$\frac{\langle \frac{\vec{\Delta}}{2}|T_i^{\mu\nu}(0)|-\frac{\vec{\Delta}}{2}\rangle}{2P^0} = M\left\{\eta^{\mu 0}\eta^{\nu 0}\left[A_i(t) + \frac{t}{4M^2}B_i(t)\right] + \eta^{\mu\nu}\left[\bar{C}_i(t) - \frac{t}{M^2}C_i(t)\right] + \frac{\Delta^\mu\Delta^\nu}{M^2}C_i(t)\right\}$$

Analogy with relativistic hydrodynamics $r = |\vec{r}|$

Anisotropic fluid

$$\Theta_i^{\mu\nu}(\vec{r}) = [\varepsilon_i(r) + p_{t,i}(r)]\,u^\mu u^\nu - p_{t,i}(r)\eta^{\mu\nu} + [p_{r,i}(r) - p_{t,i}(r)]\,\frac{r^\mu r^\nu}{r^2}$$



Isotropic pressure

$$p_i(r) = \frac{p_{r,i}(r) + 2p_{t,i}(r)}{3}$$

Pressure anisotropy

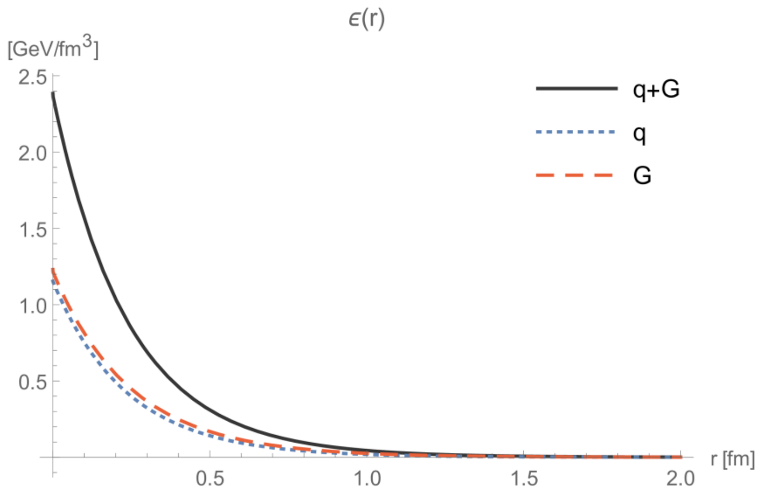
$$s_i(r) = p_{r,i}(r) - p_{t,i}(r)$$

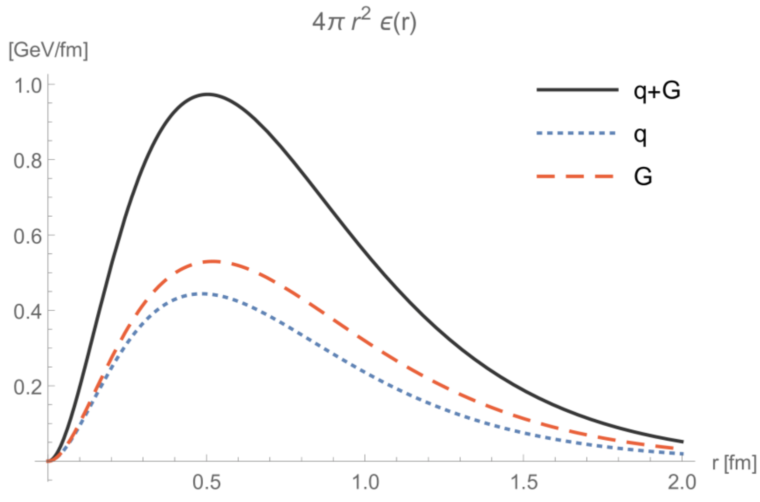
Energy distribution

[C.L., Moutarde, Trawinski, *in preparation*]

Multipole model for the GFFs

$$F(t) = \frac{F(0)}{(1 + t/\Lambda^2)^n}$$

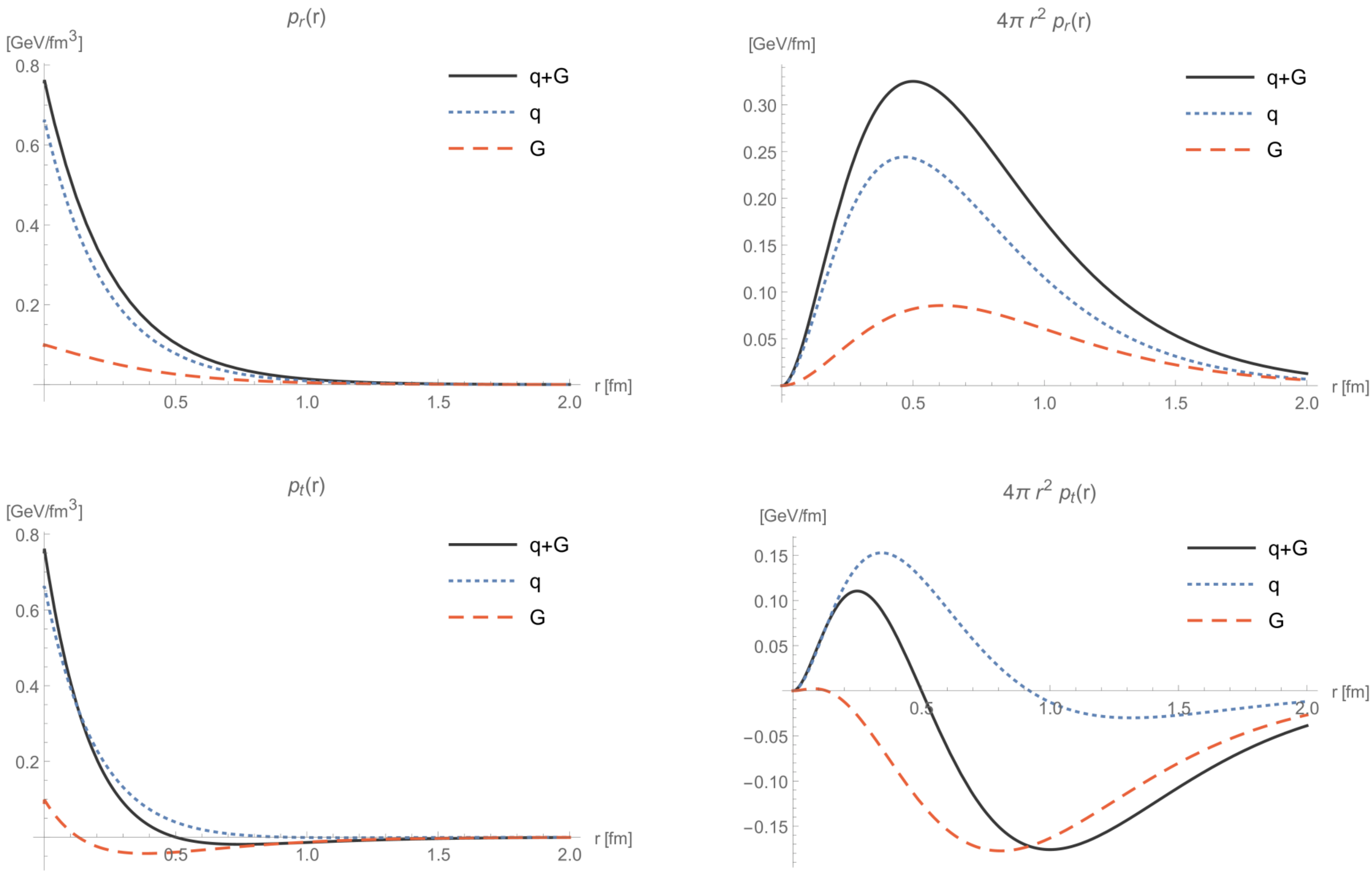




$$\sqrt{\langle r^2 \rangle_M} = 0.91\,\mathrm{fm}$$
$$\sqrt{\langle r^2 \rangle_Q} = 0.84 - 0.88\,\mathrm{fm}$$

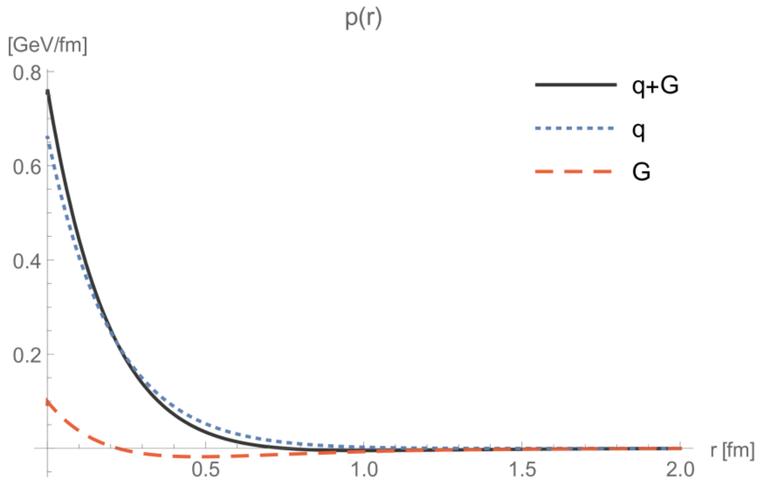
Pressure distribution

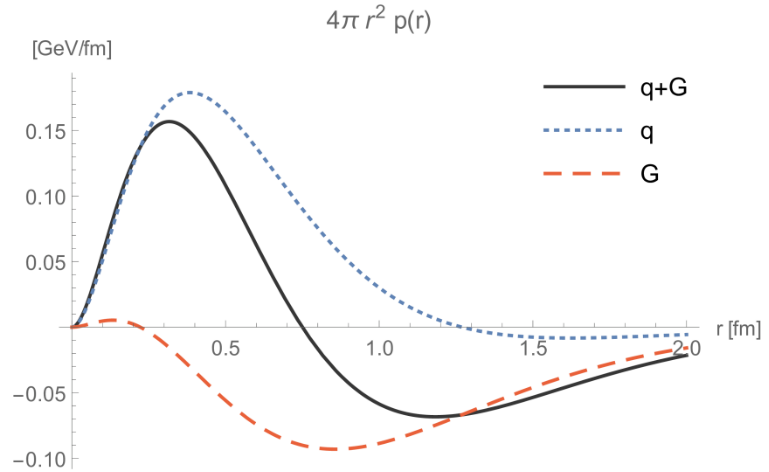
[C.L., Moutarde, Trawinski, *in preparation*]

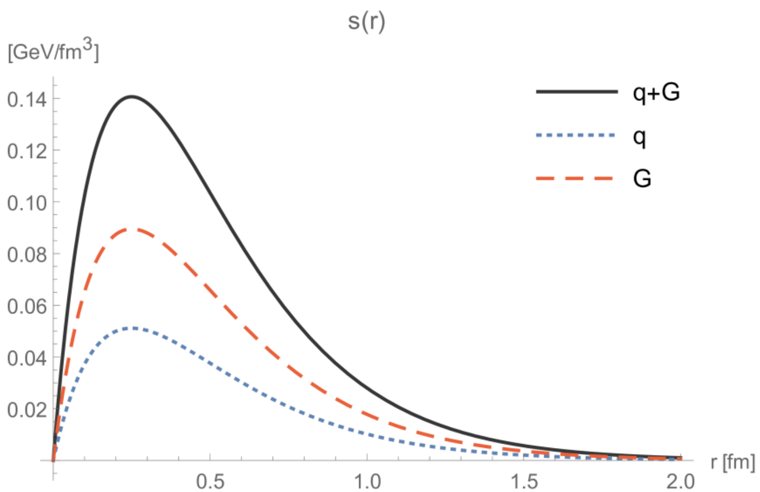


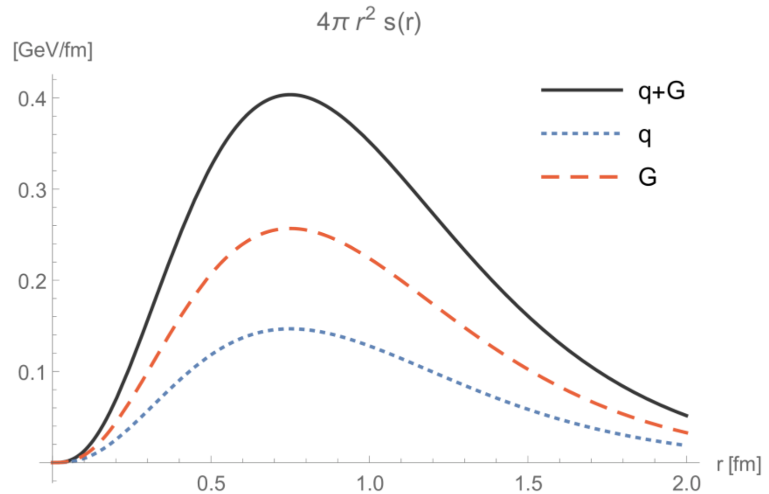
Pressure distribution

[C.L., Moutarde, Trawinski, *in preparation*]





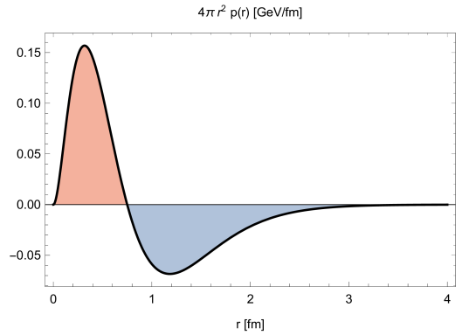




Hydrostatic equilibrium

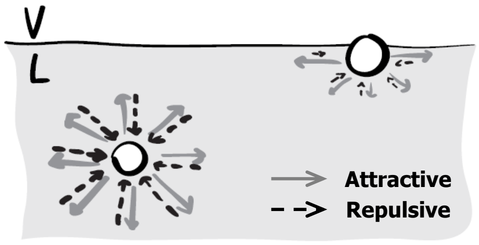
[Polyakov (2003)]
[Goeke *et al.* (2007)]
[Polyakov, Schweitzer (2018)]
[C.L., Moutarde, Trawinski, *in preparation*]

$$\nabla^i T^{ij}(\vec{r}) = 0 \quad \Longrightarrow \quad \frac{\mathrm{d}p_r(r)}{\mathrm{d}r} = -\frac{2s(r)}{r}$$



von Laue relation [von Laue (1911)]

$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0$$



Surface tension [Bakker (1928)] [Kirkwood, Buff (1949)] [Marchand *et al.* (2011)]

$$\gamma = \int \mathrm{d}r\, s(r)$$

Generalized Young-Laplace relation

[Thomson (1858)]

$$p(0) = 2\int_0^\infty \mathrm{d}r\, \frac{s(r)}{r}$$

$$s(r) \approx \gamma\, \delta(r - R) \quad \Longrightarrow \quad p(0) = \frac{2\gamma}{R}$$

Compact stars

White dwarfs, neutron stars, quark stars, strange stars, gravastars, black holes, …

Highest energy densities and strongest gravitational fields!

Tests under extreme conditions

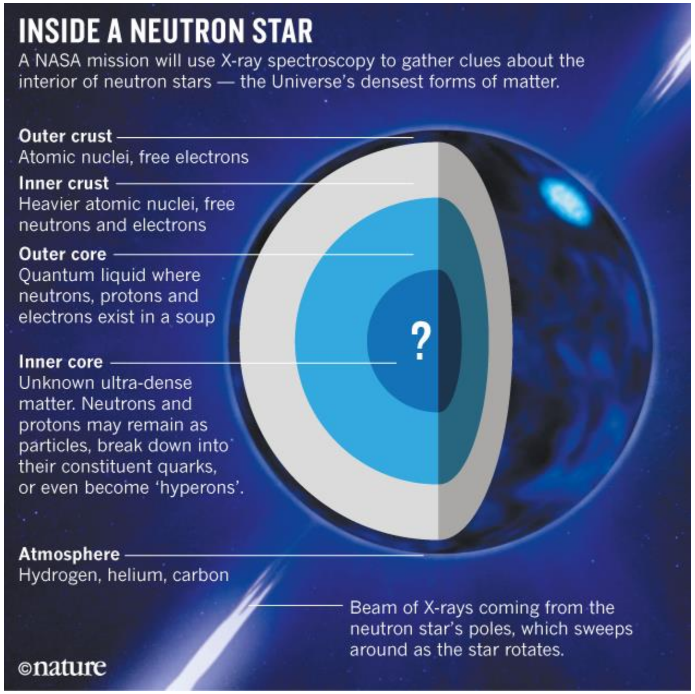
- Nuclear matter
- General relativity & alternatives

[Berti *et al.* (2015)]
[Lattimer, Prakash (2016)]

EMT is likely anisotropic

- Relativistic nuclear interactions
- Mixture of fluids of different types
- Presence of superfluid
- Existence of solid core
- Phase transitions
- Presence of magnetic field
- Viscosity
- …

[Ruderman (1972)]
[Canuto (1974)]
[Bowers, Liang (1974)]
[Herrera, Santos (1997)]



$M \sim 1.4\, M_\odot$ $\qquad M_\odot = 2 \times 10^{33}\,\mathrm{g}$

$R \sim 10\,\mathrm{km}$

$\rho \sim 3\,\rho_0$ $\qquad \rho_0 = 2.8 \times 10^{14}\,\mathrm{g/cm^3}$

$g \sim 2.4 \times 10^{12}\,\mathrm{m/s^2}$

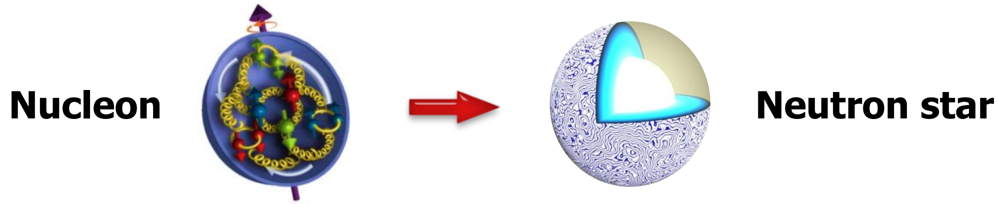
[Potekhin (2010)]

$M_p \sim 1.67 \times 10^{-24}\,\mathrm{g}$

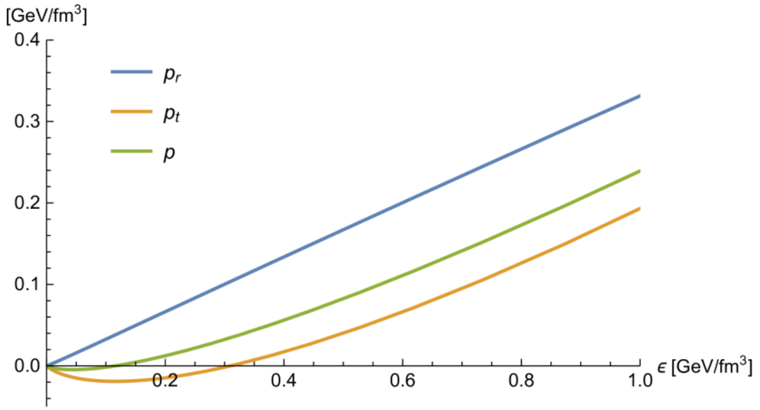
$R_p \sim 0.84\,\mathrm{fm}$

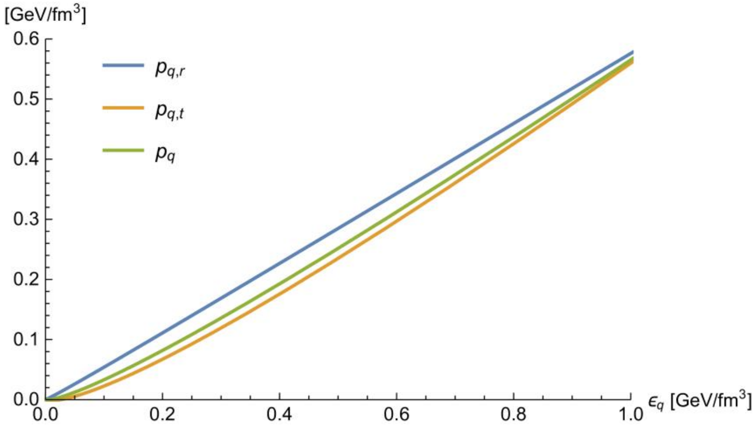
$\rho_p \sim 2.4\,\rho_0$

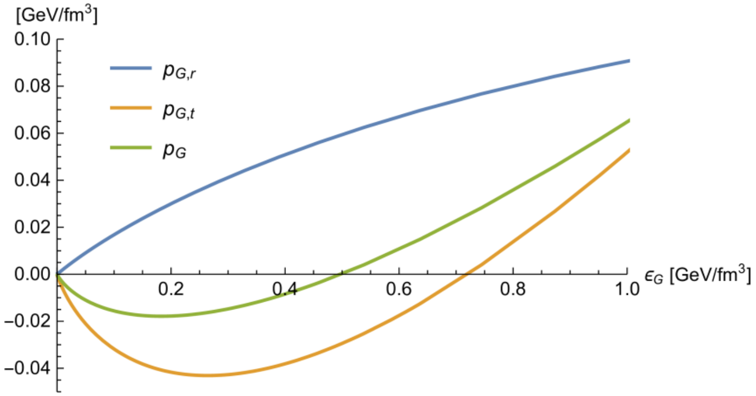
What can we learn?



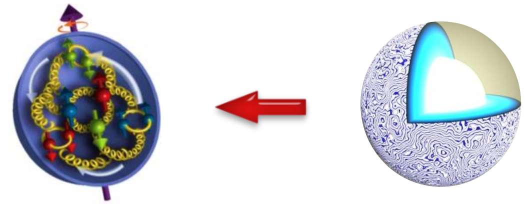
Equation of state







What can we learn?

Nucleon  **Neutron star**

Stability constraints

[Wald (1984)]
[Herrera, Santos (1997)]
[Poisson (2004)]
[Abreu, Hernandez, Nunez (2007)]
[Hawking, Ellis (2011)]

Mechanical regularity

(i) $\varepsilon(0) < \infty$, $p(0) < \infty$ and $s(0) = 0$;

(ii) $\varepsilon(r) > 0$ and $p_r(r) > 0$;

(iii) $\frac{\mathrm{d}\varepsilon(r)}{\mathrm{d}r} < 0$ and $\frac{\mathrm{d}p_r(r)}{\mathrm{d}r} < 0$.

Speed of sound

(iv) $0 \leq v_{sr}^2(r) \leq 1$ and $0 \leq v_{st}^2(r) \leq 1$;

(v) $|v_{st}^2(r) - v_{sr}^2(r)| \leq 1$;

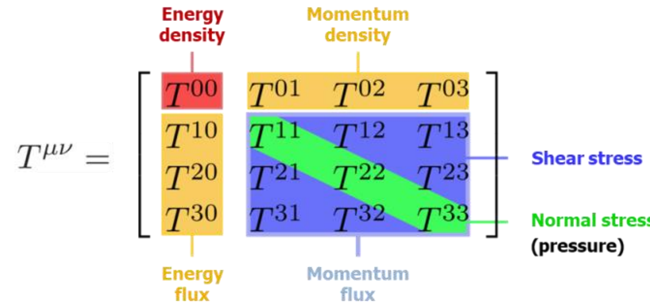
(vi) $\Gamma(r) = \frac{\varepsilon(r)+p_r(r)}{p_r(r)}\, v_{sr}^2 > \frac{4}{3}$.

Energy conditions

$$\varepsilon(r) + p_i(r) \geq 0,$$
$$\varepsilon(r) + p_i(r) \geq 0 \quad \text{and} \quad \varepsilon(r) \geq 0,$$
$$\varepsilon(r) + p_i(r) \geq 0 \quad \text{and} \quad \varepsilon(r) + 3\,p(r) \geq 0,$$
$$\varepsilon(r) \geq |p_i(r)|,$$

Summary

- **Mass, spin and pressure all encoded in EMT**



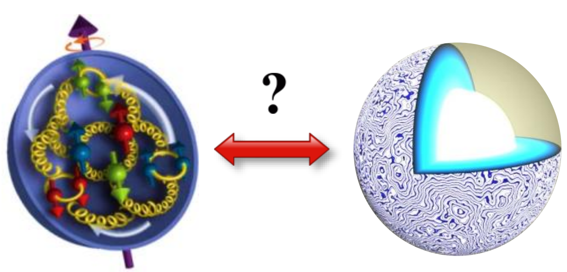
- **Nucleon energy density and pressure are extremely high**

  → **Large anisotropy**

- **Same ballpark as interior of compact stars**

- **Exciting cross-talk between hadronic physics and neutron star physics!**

  → **Equation of state, stability contraints, …**



Backup slides

Gravitational form factors

Mellin moment of twist-2 vector GPDs $\langle p', s'|T^{++}(0)|p, s\rangle$

$$\int \mathrm{d}x\, x\, H(x, \xi, t) = A(t) + 4\xi^2 C(t)$$

$$\int \mathrm{d}x\, x\, E(x, \xi, t) = B(t) - 4\xi^2 C(t)$$

[Ji (1996)]

Poincaré covariance $\langle p', s'|T_q^{[\alpha\beta]}(0)|p, s\rangle = -i\Delta_\mu \langle p', s'|S_q^{\mu\alpha\beta}(0)|p, s\rangle$

$$D_q(t) = -G_A^q(t)$$

[C.L., Mantovani, Pasquini (2018)]

EMT trace

[Ji (1995)]
[C.L. (2018)]

$$\langle p', s'|G^2(0)|p, s\rangle, \quad \langle p', s'|\overline{\psi}(0)\psi(0)|p, s\rangle \quad \longrightarrow \quad \bar{C}(t)$$

Recent Lattice QCD results

Mass

Spin

Momentum

[Yang *et al.*, arXiv:1808.08677]

[Alexandrou *et al.*, arXiv:1807.11214]

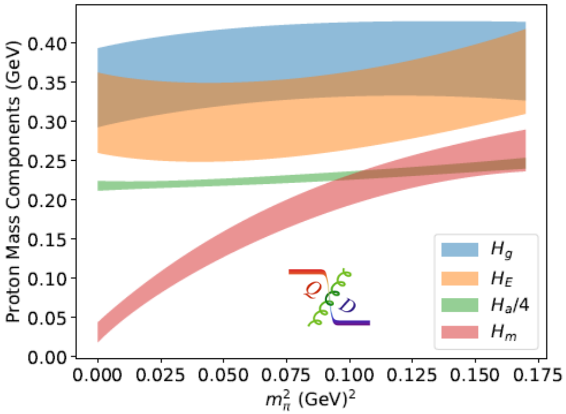


FIG. 3. The valence pion mass dependence of the proton mass decomposition in terms of the quark condensate ($\langle H_m \rangle$), quark energy $\langle H_E \rangle$, glue field energy $\langle H_g \rangle$ and trace anomaly $\langle H_a \rangle/4$.

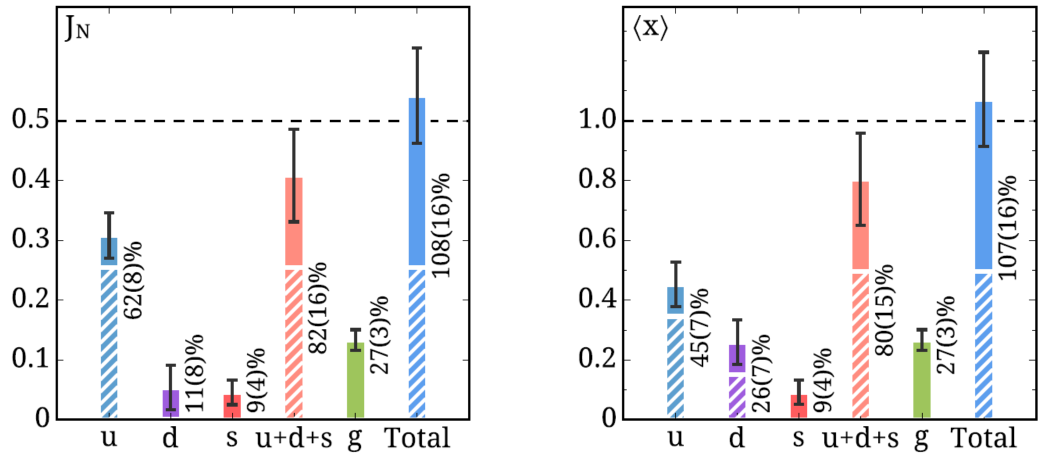


Figure 2: Left: Nucleon spin decomposition. Right: Nucleon momentum decomposition. The striped segments show valence quark contributions (connected) and the solid segments the sea quark and gluon contributions (disconnected). Results are given in $\overline{\text{MS}}$-scheme at 2 GeV.