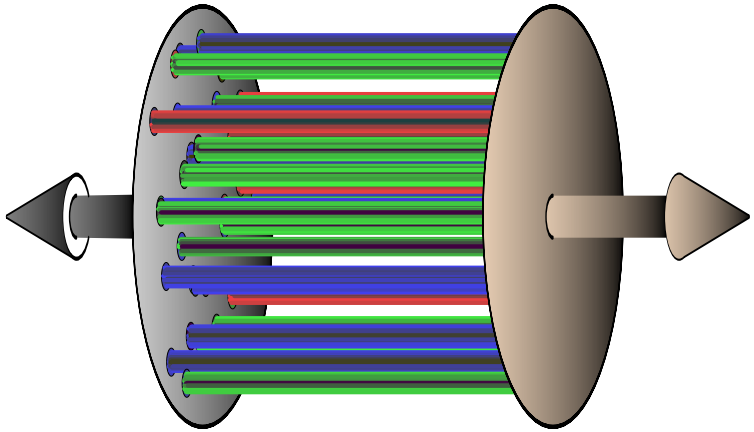
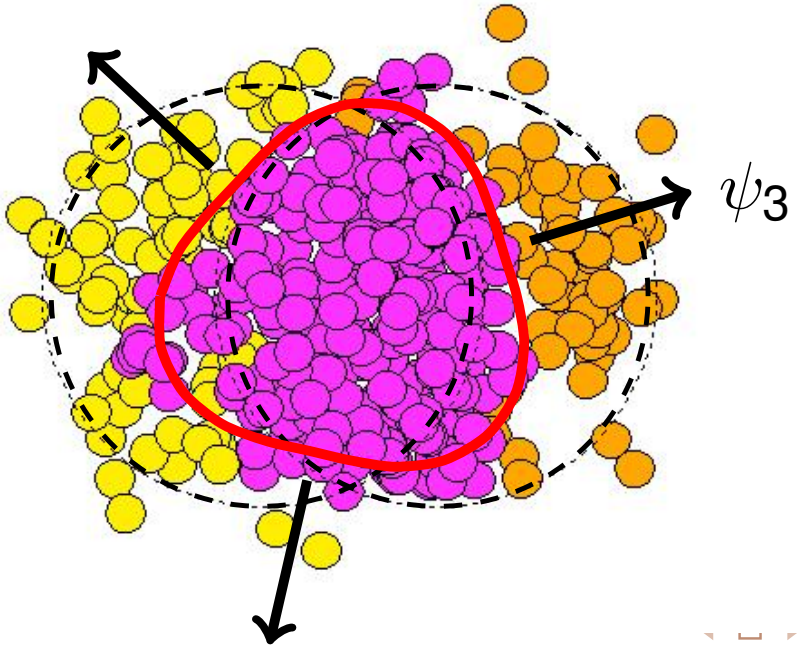
# *Nucleus-nucleus collisions: Glasma, Flow & Thermalization*

**Edmond Iancu**
Institut de Physique Théorique de Saclay





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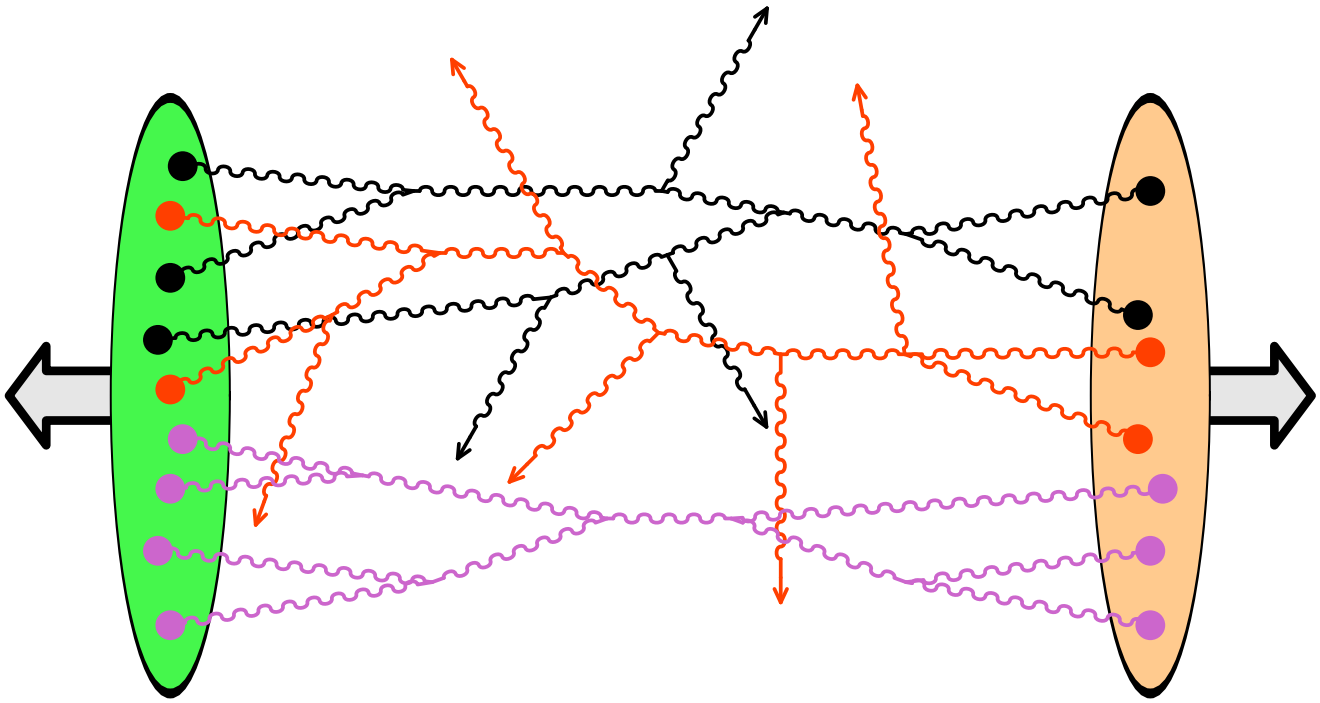

- "Dense–dense scattering" : much more complicated !
- Non–linear effects enter at all levels
  - in both incoming wavefunctions: gluon saturation
  - in the scattering process: multiple interactions
  - in the partonic medium created by the early scattering: final–state interactions

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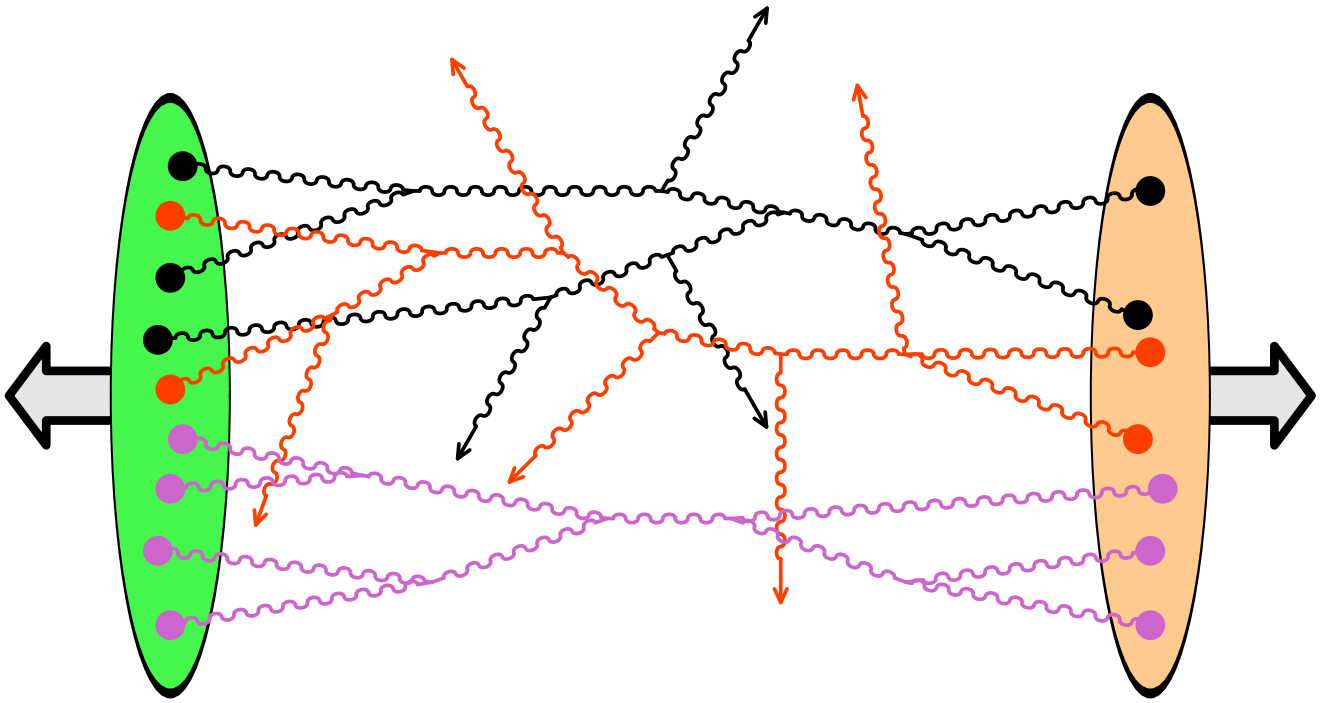

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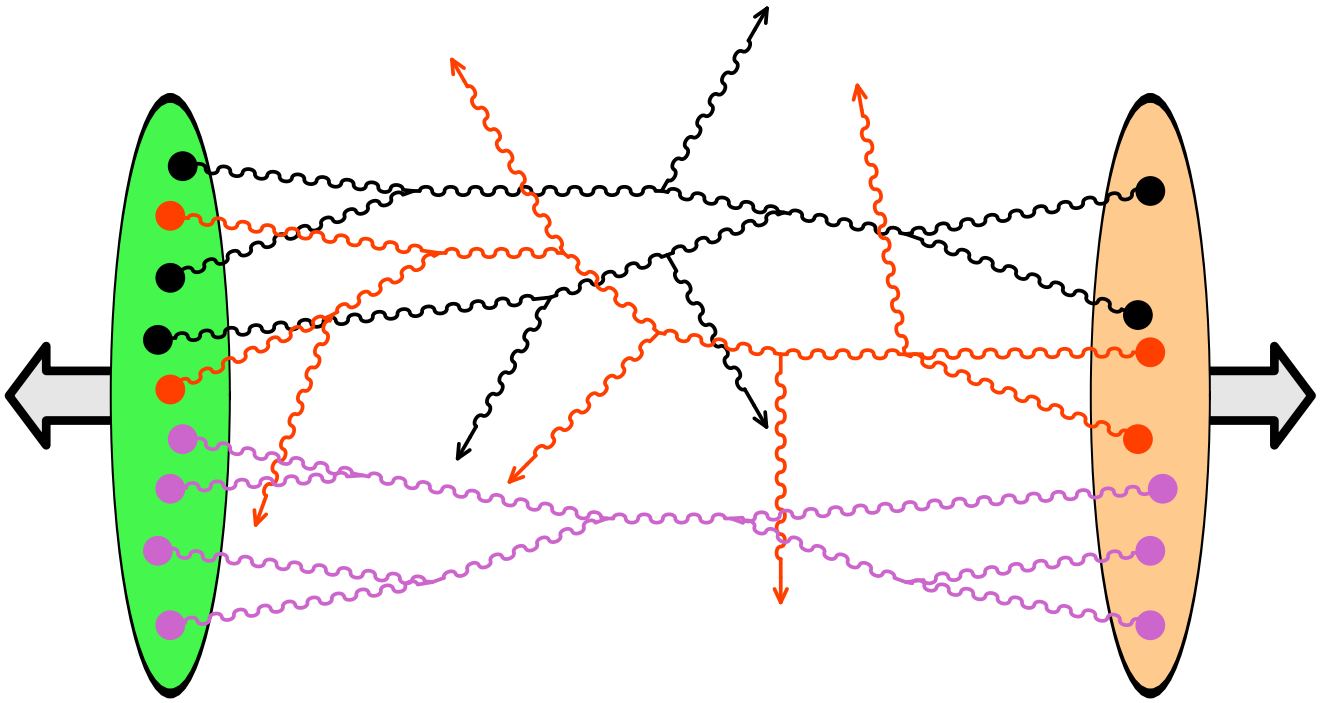

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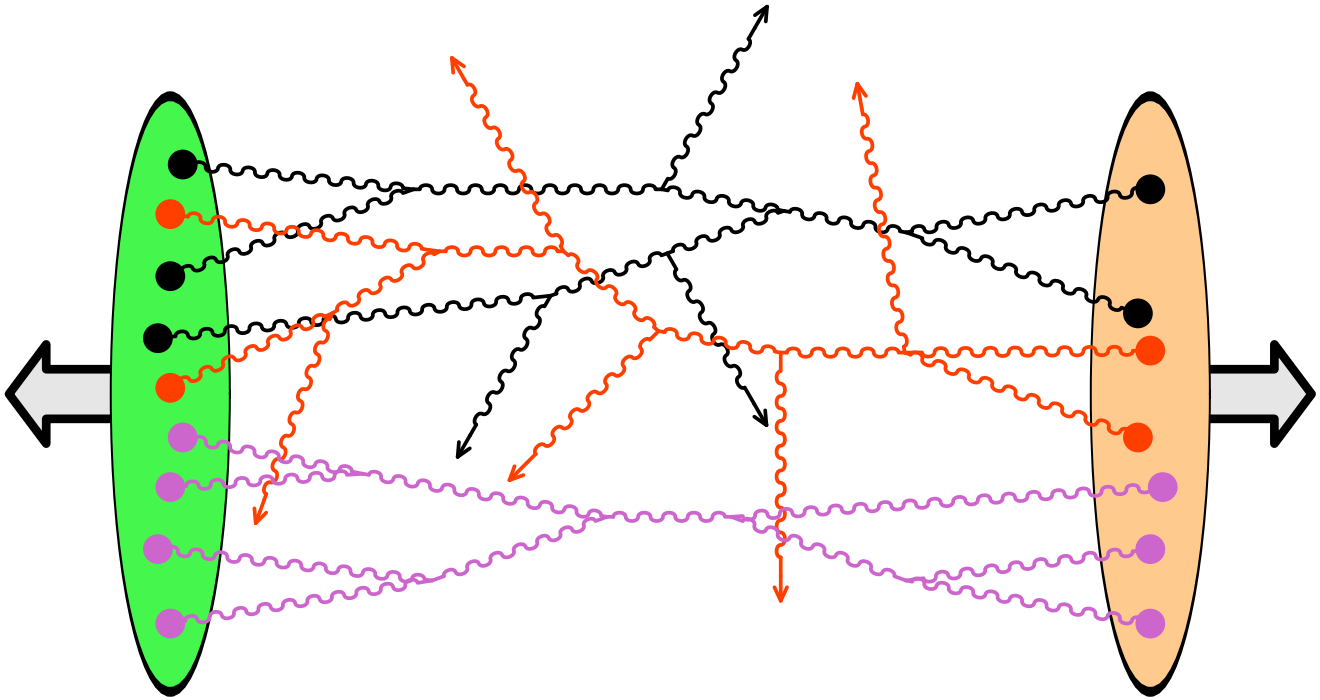

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  - kinetic theory, hydrodynamics, quark-gluon plasma, ...

# CGC factorization for $AA$ collisions

- Numerically solve classical YM equations with 2 sources (2D lattice)

$$D_\nu F^{\nu\mu}(x) \,=\, \delta^{\mu+}\rho_1(x) \,+\, \delta^{\mu-}\rho_2(x)$$

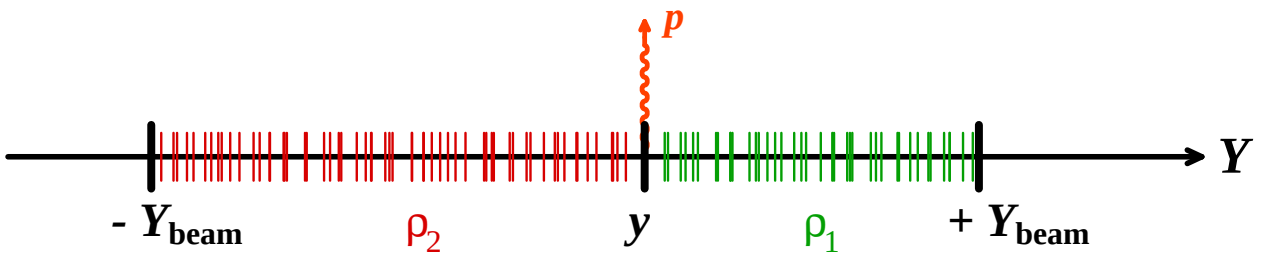
- Decompose the classical field $A_a^\mu$ in Fourier modes

  ▷ gluon spectrum for given configurations $\rho_1$ and $\rho_2$ ("event-by-event")

- Average over $\rho_1$ and $\rho_2$ using the CGC distributions of the nuclei

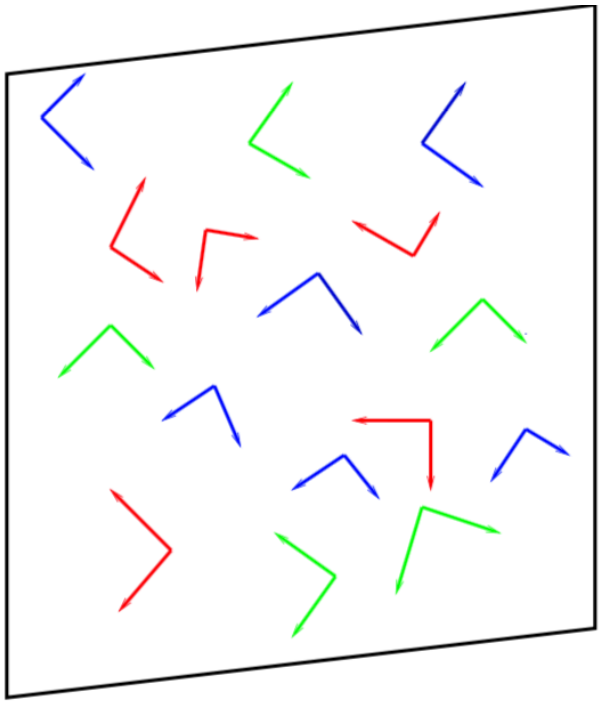
$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2p_\perp} \right\rangle \,=\, \int [\mathcal{D}\rho_1\mathcal{D}\rho_2]\, W_{Y_{\mathrm{beam}-Y}}[\rho_1]\, W_{Y_{\mathrm{beam}+Y}}[\rho_2]\, \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2p_\perp}\bigg|_{\mathrm{class}}$$

  ▷ JIMWLK evolution from $Y_{\mathrm{beam}}$ up to the rapidity $Y$ of the produced gluon



# The color field of a single nucleus

- What are the chromo-electric and magnetic fields created by a ultrarelativistic nucleus ? ($\rho(x) \propto \delta(x^+)$ for a left mover)
  - non-Abelian generalization of the Liénard-Wiechert potentials
  - Weiszäcker-Williams fields describing quasi-real photons/gluons



$$\boldsymbol{E}_a \perp \boldsymbol{B}_a \perp \boldsymbol{z}$$

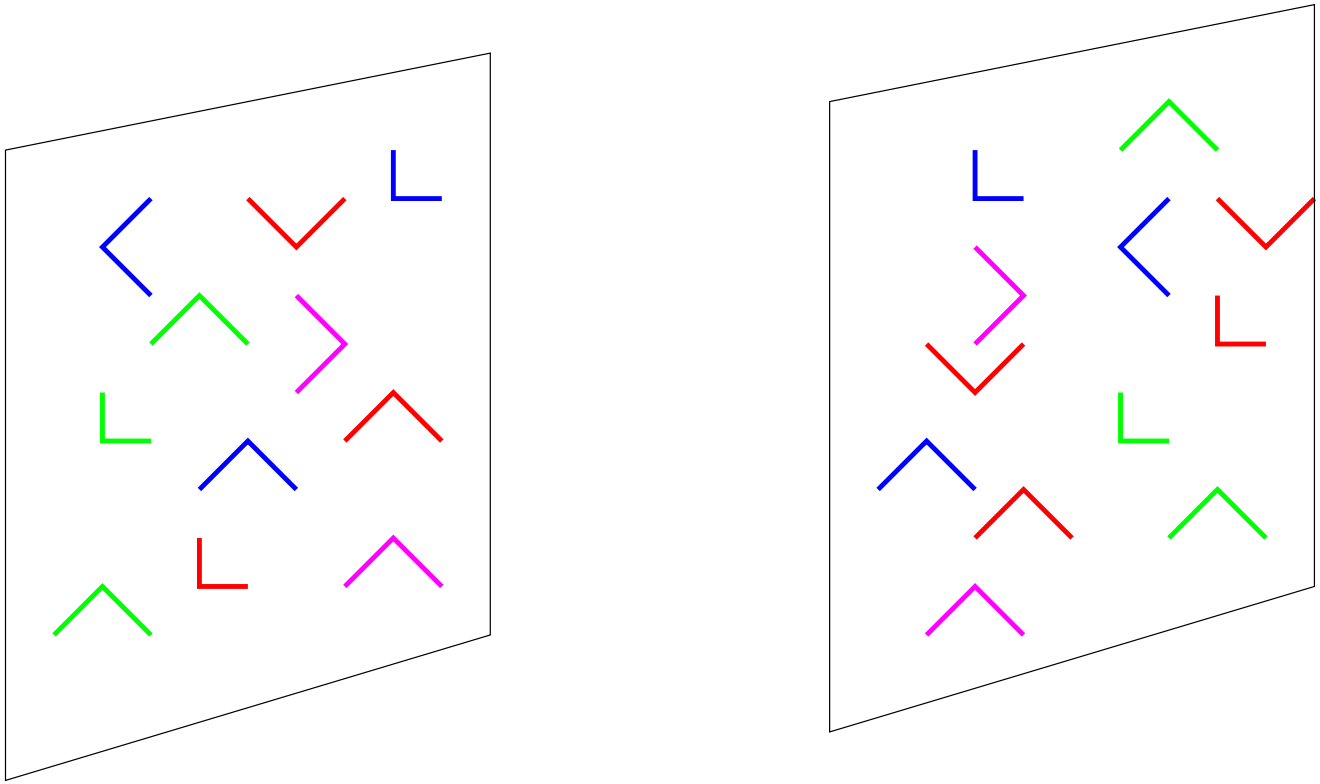
$$\boldsymbol{E}_\perp \cdot \boldsymbol{B}_\perp = 0\,, \quad |\boldsymbol{E}_\perp| = |\boldsymbol{B}_\perp| \sim \frac{1}{g}$$

- transverse polarizations
- chromo-electromagnetic waves
- Lorentz contraction: $\propto \delta(x^+)$

- Fields vary over a distance $\sim 1/Q_s \Longrightarrow$ gluons typically have $k_\perp \sim Q_s$
- Fields have strength $\sim 1/g \Longrightarrow$ gluons have occupation numbers $\sim 1/\alpha_s$

# The scattering between two color sheets

- Prior to scattering: purely transverse fields, localized near the 2 light-cones



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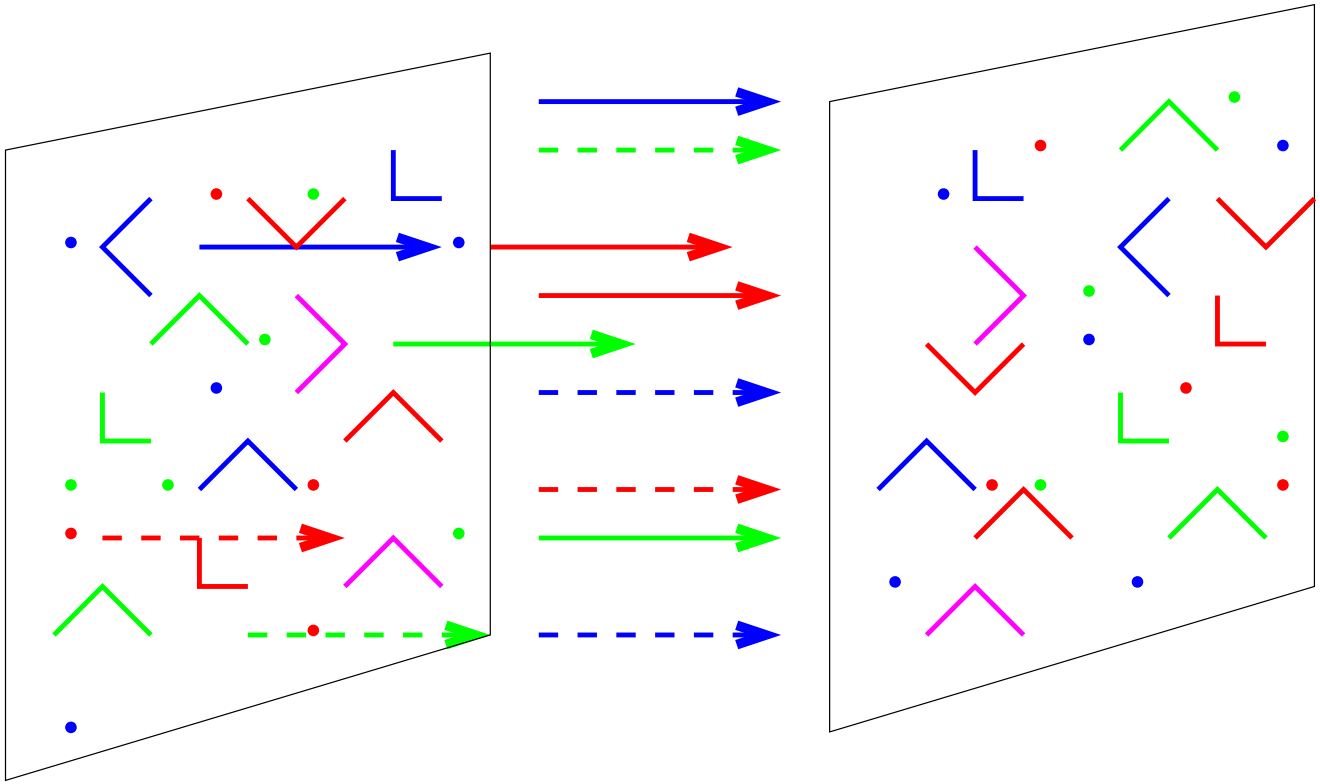
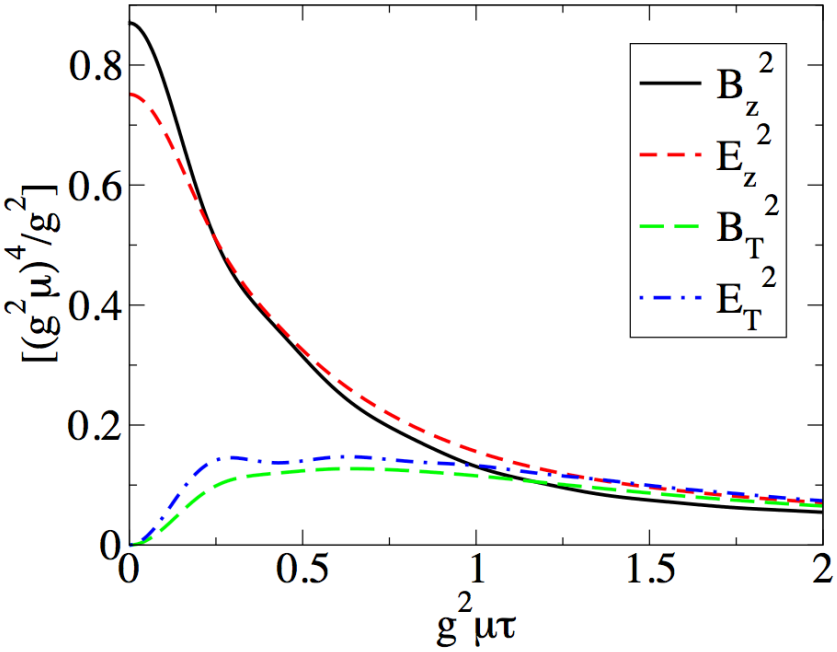
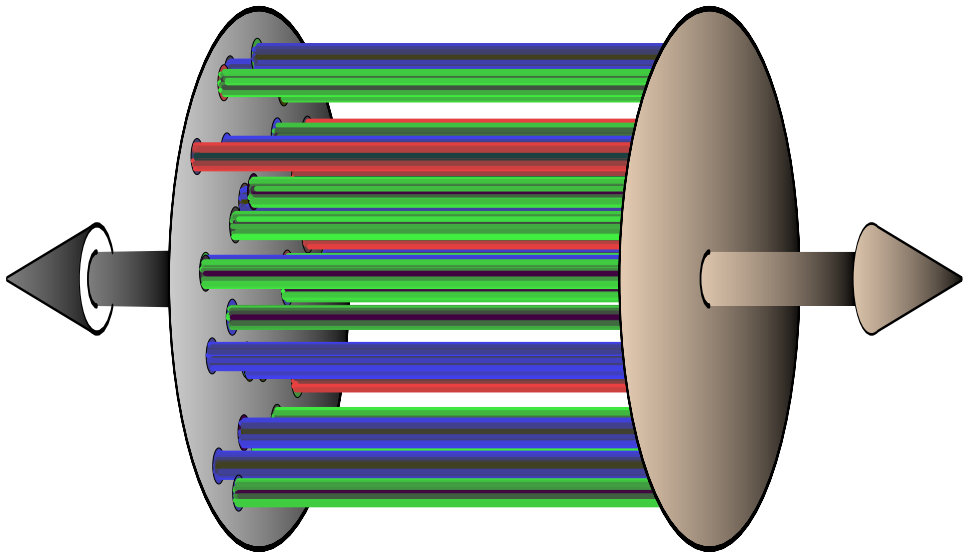


- During collision, mutual color rotations induce color charges on the sheets
  - longitudinal chromo-electric and chromo-magnetic fields
  - color strings (flux tubes) with typical transverse size $1/Q_s$

# Glasma *(McLerran and Lappi, 06)*

- The 'valence' charges of the 2 nuclei rapidly separate from each other
- The color field between the 2 recessing nuclei: 'glasma' ('glass' + 'plasma')

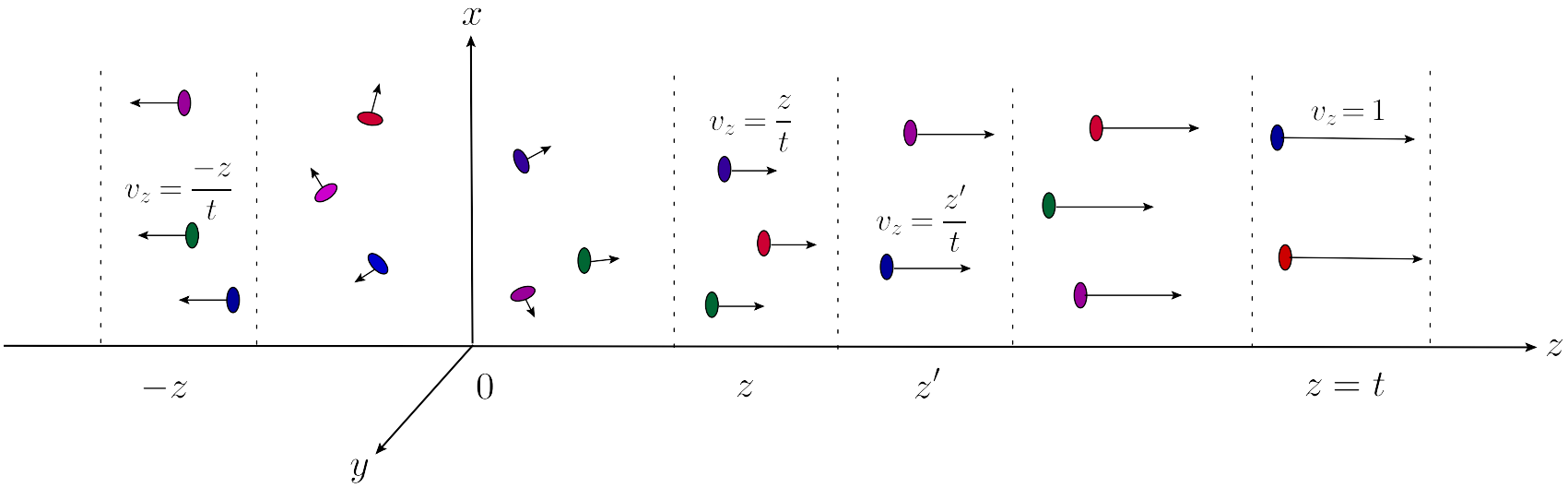


- After a time $\tau \sim 1/Q_s$, the transverse fields are regenerated
- By that time, the partonic system becomes dilute (field strengths become of $\mathcal{O}(1)$), due to longitudinal expansion
- Fourier mode decomposition $\Longrightarrow$ gluon production

# Longitudinal expansion

- Gluons liberated by the collision at $t = 0$ have transverse momenta $k_\perp \sim Q_s$ and generic longitudinal momenta $|k_z| \sim x_1 P_1^+$ or $|k_z| \sim x_2 P_2^-$
- After a time $t \sim 1/Q_s$, they separate from each other along the $z$ axis
  - particles which at time $t$ are located at $z$ have a velocity $v_z = \frac{z}{t}$




- In any slice of $z$, the distribution in the transverse plane is roughly isotropic
- But the 3-dimensional distribution in momentum is highly anisotropic

# Rapidity correlations

- Consider pairs of particles in the final state with different rapidities
  - hadrons which propagate at different angles w.r.t. the collision axis

$$\eta = \frac{1}{2}\ln\frac{p+p_z}{p-p_z} = \frac{1}{2}\ln\frac{1+v_z}{1-v_z} = -\ln\tan\frac{\theta}{2}$$

- Construct the 2-hadron correlation in $\eta$ and $\boldsymbol{p}_\perp$ (cf. talk by Jan Fiete):

$$\mathcal{C}(\Delta\phi,\Delta\eta) \equiv \frac{\mathrm{d}N_{\mathrm{pair}}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

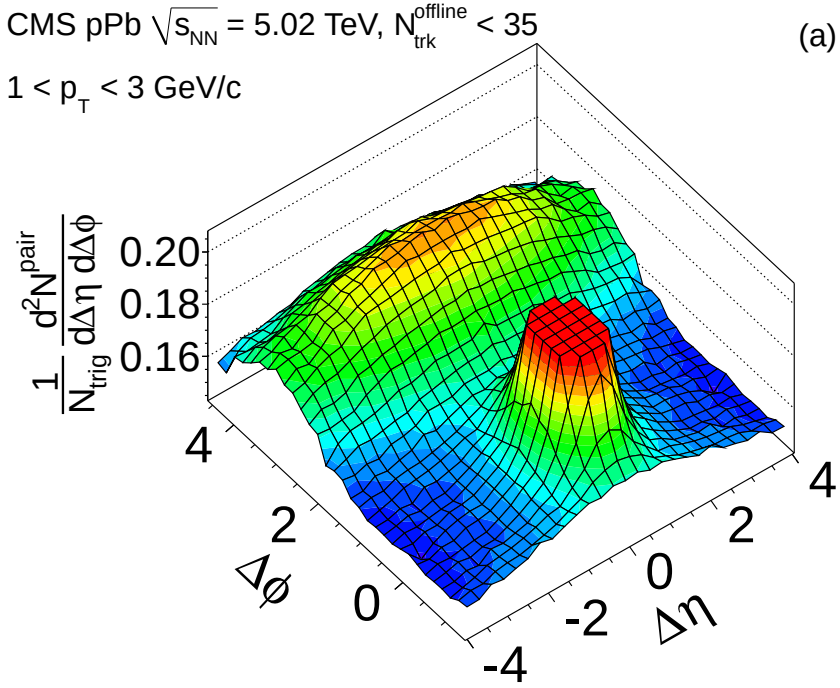
  - interested in the distribution in $\Delta\eta = \eta_1 - \eta_2$ and in $\Delta\phi = \phi_1 - \phi_2$

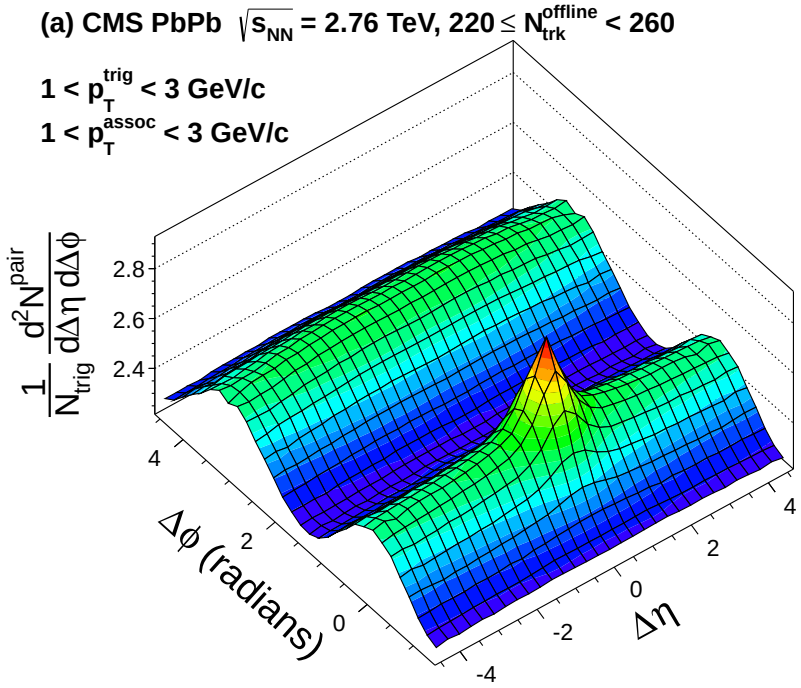
- Large $\Delta\eta > 1 \Longrightarrow$ particles with very different longitudinal velocities
  - they separate from each other after a time $\tau \sim 1/Q_s \simeq 0.2$ fm
  - their correlations must have been built at smaller times $\tau \lesssim 1/Q_s$

- Long-range correlations in rapidity teach us about the early stages/glasma

# Collective flow in $AA$ collisions (*cf. lecture by Jan Fiete*)

- Left: p+Pb collisions: "trivial" correlations associated with jets
  - a peak around $\Delta\eta = 0$ and $\Delta\phi = 0$: particles from a same jet
  - a plateau in $\Delta\eta$ narrow in $\Delta\phi$ around $\Delta\phi = \pi$: recoiling jets

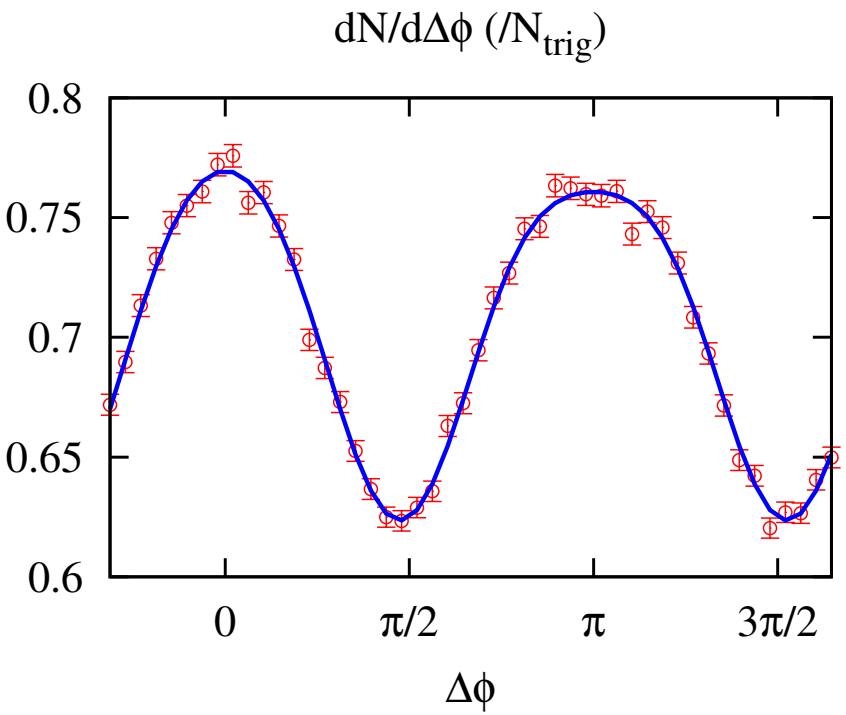


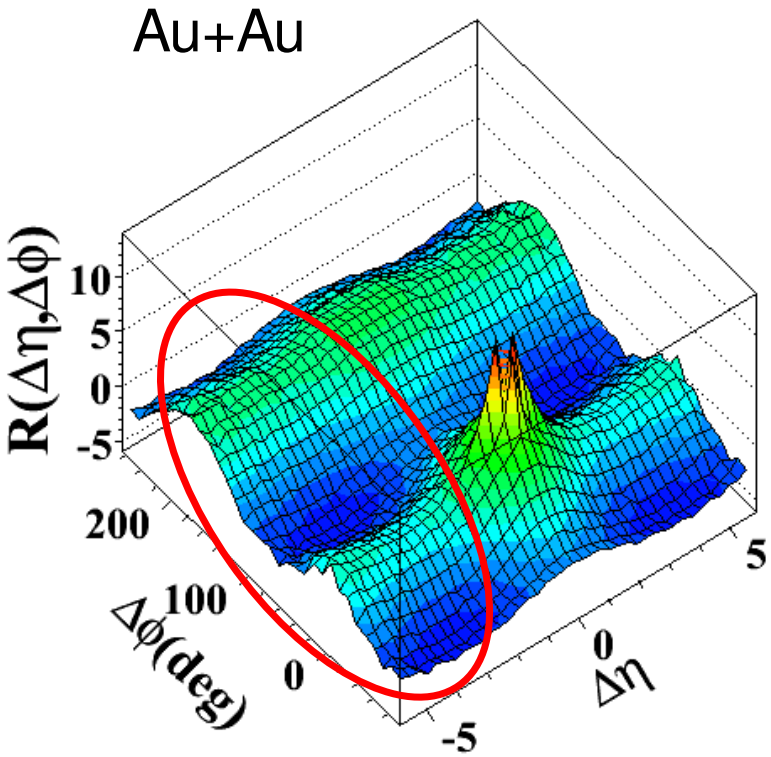




- Right: Pb+Pb collisions: long-range correlations in $\Delta\eta$ narrow in $\Delta\phi$ both on the "near side" ($\Delta\phi \simeq 0$) and on the "away side" ($\Delta\phi \approx \pi$)

# Elliptic flow

- What is the origin of the double peak structure ($\Delta\phi = 0$ and $\pi$) ?

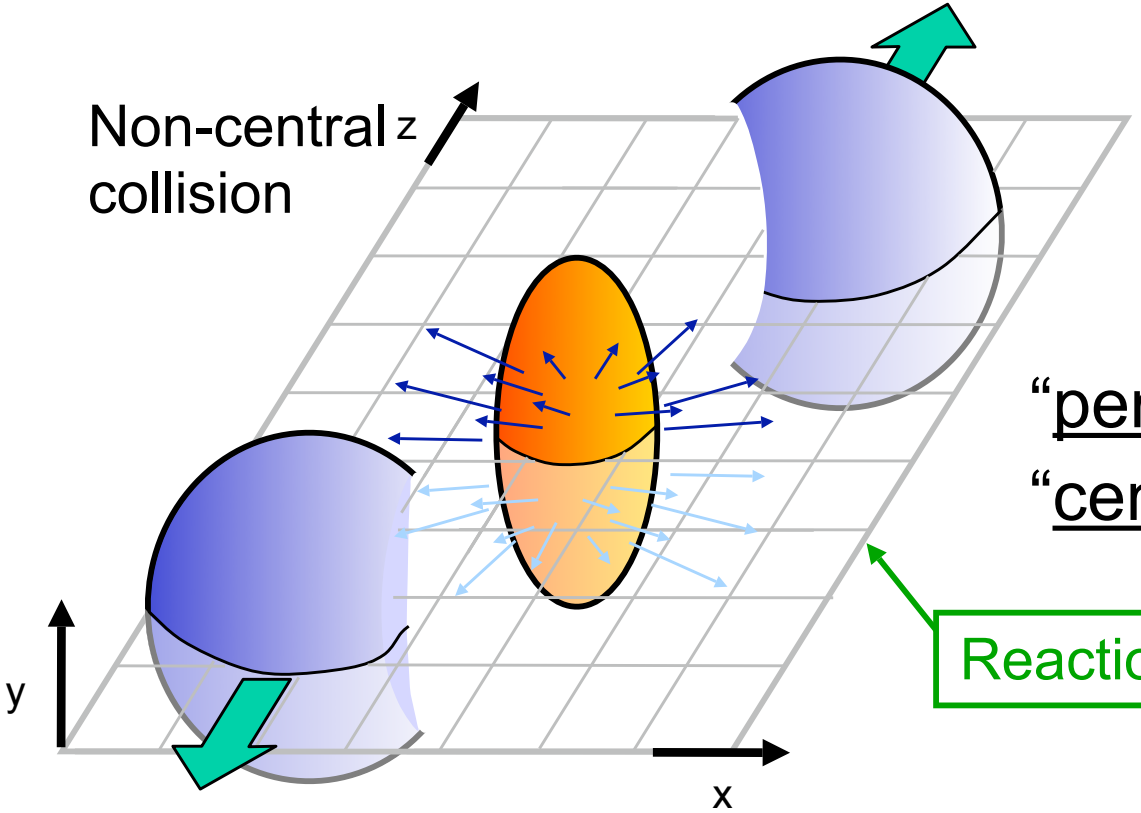



$$\mathcal{R} \equiv \frac{\langle N_1\, N_2\rangle - \langle N_1\rangle\,\langle N_2\rangle}{\langle N_1\rangle\,\langle N_2\rangle} \propto v_2^2\,\cos\left(2\Delta\phi\right)$$

- Maximum at $\Delta\phi = 0$ or $\pi$; minimum at $\Delta\phi = \pi/2$: elliptic flow

# The geometry of a HIC



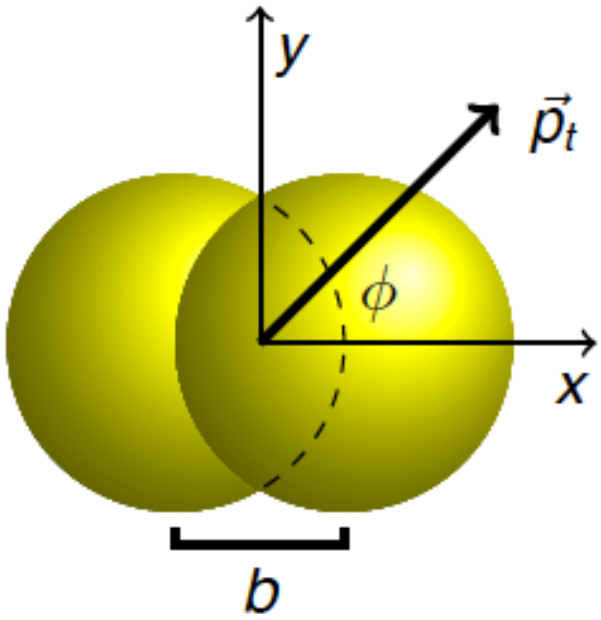

"peripheral" collision ($b \sim b_{max}$)

"central" collision ($b \sim 0$)

Reaction plane

Number of participants ($N_{part}$): number of incoming nucleons (participants) in the overlap region

# Elliptic flow $v_2$



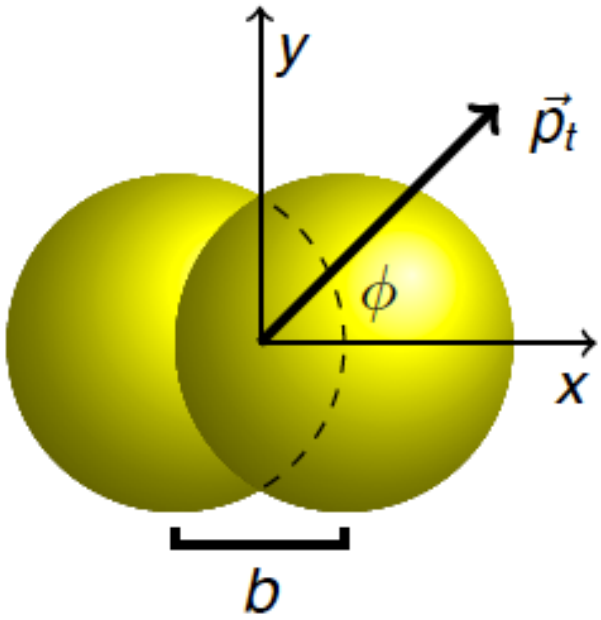

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2\phi$$

$v_2$ : the 'coefficient of the elliptic flow'

- Non-central $AA$ collision: impact parameter $b_\perp > 0$
- The interaction region is (roughly) elliptic
- Pressure gradient is larger along the smaller axis ($x$)
- Fluid velocity is proportional to the pressure gradient
- Particle emerge predominantly parallel to the fluid velocity

$\Longrightarrow$ the particle distribution is not axially symmetric !

# Elliptic flow $v_2$

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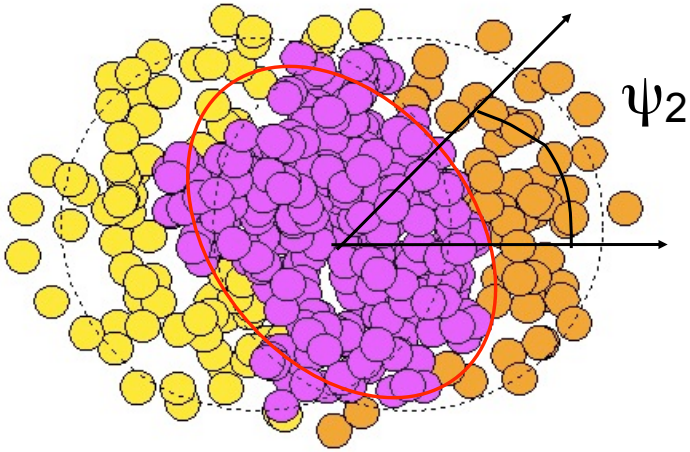
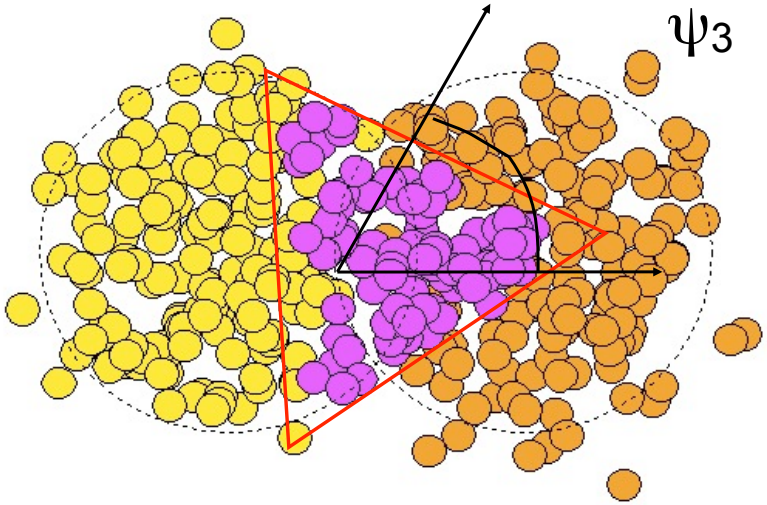
- Initial anisotropy in coordinate space yields final anisotropy in momentum

# Granularity and fluctuations



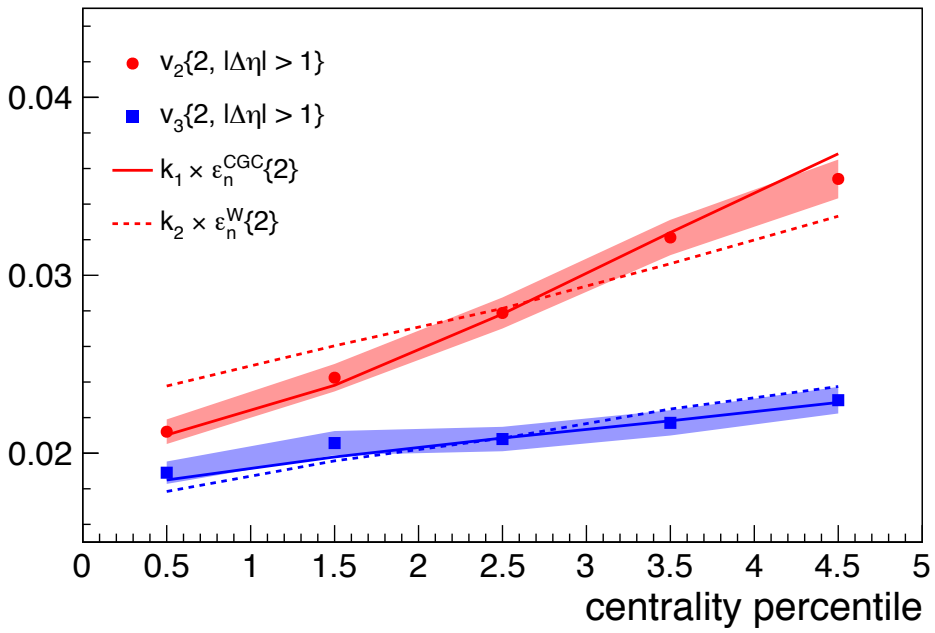

- Nucleons are randomly distributed inside a nucleus.
- In some events, the shape of the interaction (overlap) region can be quite different from an ellipse
- Then one speaks about triangular flow (or higher harmonics)

$$\frac{\mathrm{d}N}{\mathrm{d}\phi} \propto 1 + 2v_2 \cos 2(\phi - \Psi_2) + 2v_3 \cos 3(\phi - \Psi_3) + \ldots$$

- The small disks need not be nucleons: they can also be saturation domains
  - $\Delta x_\perp \sim 1/Q_s \sim 0.2\,\mathrm{fm} \ll R_\mathrm{N} \sim 1$ fm

# Centrality dependence for $v_2$ and $v_3$

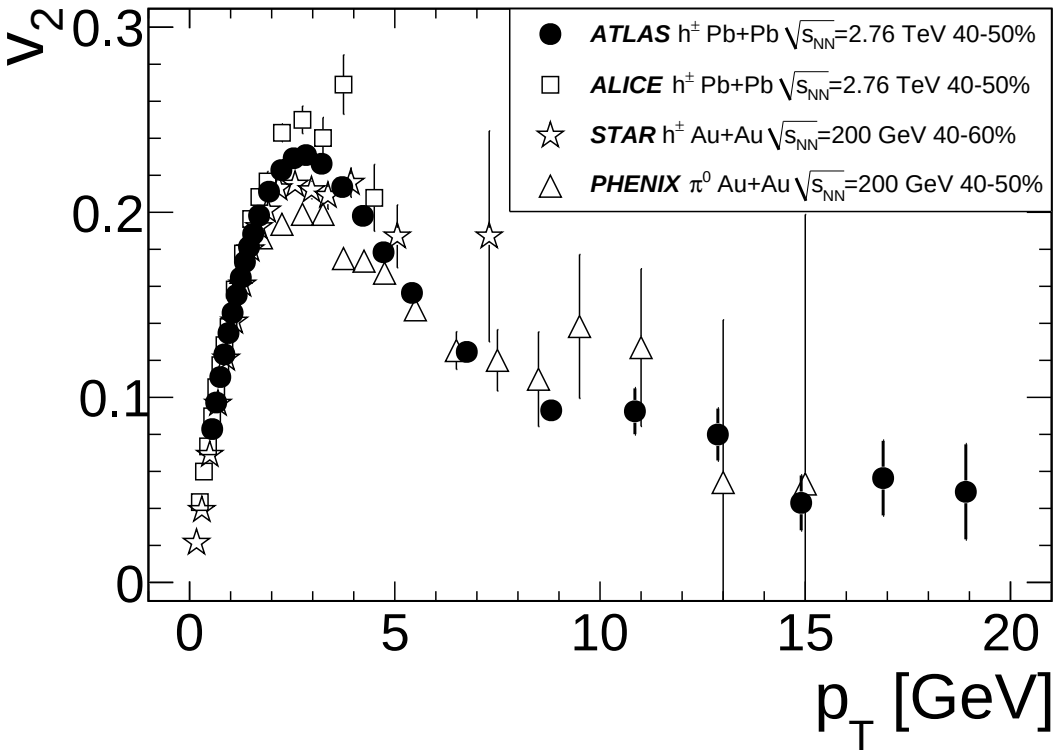
*ALICE, arXiv:1105.3865*




$$\varepsilon_2 = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$

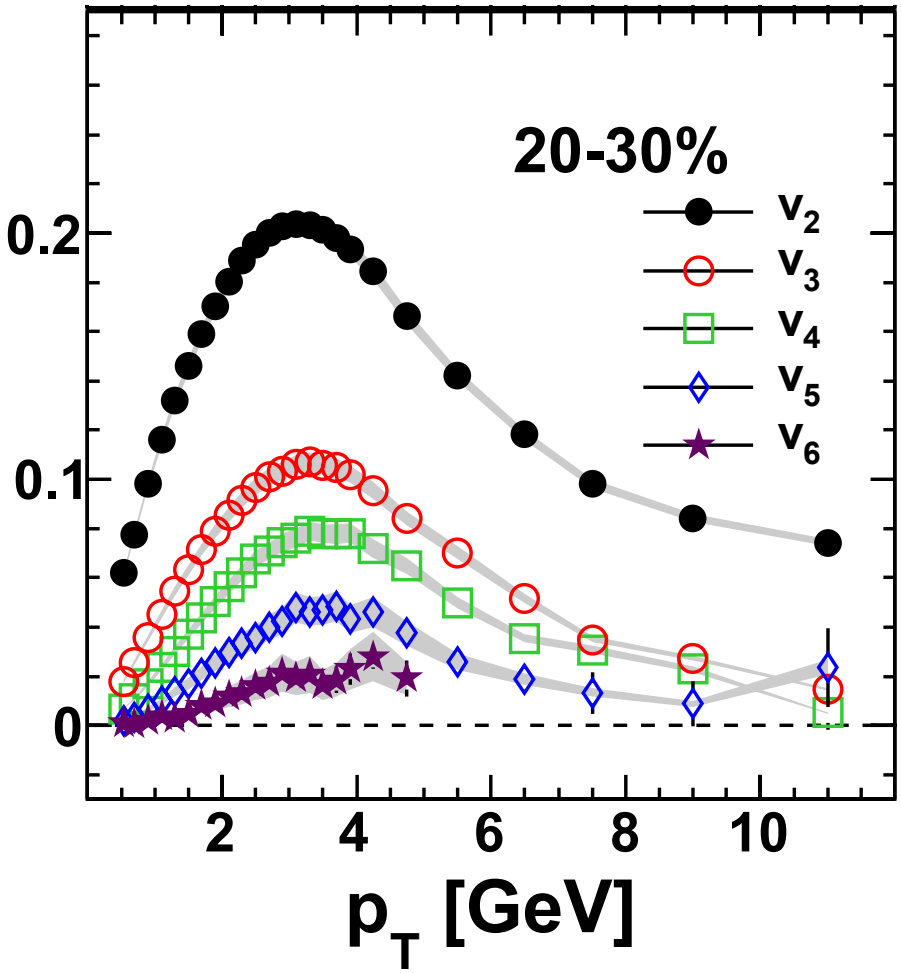
- Ultra-central collisions $0 \div 5\%$: the more central, the smaller $v_2$
  - central collisions: the interaction region is axially symmetric
  - one expects $v_2$ to be proportional to the eccentricity
  - CGC calculations of the initial $\varepsilon_2$ compare better than just a random distribution of "wounded" nucleons (Glauber)
- $v_3$ is much less sensitive to the centrality: controlled by fluctuations

# Momentum dependence for $v_2$




- $v_2$ first rises up to $3 \div 4$ GeV, then it decreases again
  - the flow is collective motion which contributes to $p_\perp$
  - sufficiently fast particles cannot be driven by the flow
- The high-$p_\perp$ tail: natural when fluctuations occur on smaller scales: $1/Q_s \ll R_N$

# $p_\perp$ dependence for $v_n$, $n = 2 - 6$ (ATLAS)




- Similar $p_\perp$ dependence for all $n$: rise up to 3-4 GeV, then fall

# Hydrodynamics in a nut shell

- To describe flow, it is natural to use hydrodynamics
  - transport of conserved quantities (energy, momentum, electric and baryonic charges ...) via collective motion with fluid velocity $\boldsymbol{v}$
- This is perhaps best introduced by comparing with thermodynamics
- Thermodynamics is about a system in global thermal equilibrium
  - pressure ($P$), temperature ($T$), chemical potential ($\mu$) are independent of time and uniform throughout the whole volume $V$
- Hydrodynamics is about local thermal equilibrium
  - $P$, $T$, $\mu$ and $\boldsymbol{v}$ can vary with space and time ...
  - ... but they vary so slowly that one can still assume thermal equilibrium to hold locally, in the neighbourhood of any point
  - equations of motion for $P(x)$, $T(x)$, $\mu(x)$, $\boldsymbol{v}(x)$
- This holds when the typical scale for space–time variations ('system size $R$') is much larger than the mean free path $\ell$ between two successive collisions

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- Gradient expansion in powers of $K \equiv \ell/L \ll 1$ (Knudsen number)
- In heavy ion collisions, $L \sim R_A$: transverse size of the nucleus

# Ideal Hydrodynamics

- Hydro equations of motion: the conservation laws

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu J_B^\mu = 0$$

- $T^{\mu\nu}$ (energy-momentum tensor) and $J_B^\mu$ (baryonic current) :
  - fluid 4-velocity: $u^\mu(x) = \gamma(1, \mathbf{v}),\ \gamma = 1/\sqrt{1-v^2}$
  - energy density $\epsilon(x)$, pressure $P(x)$, baryon density $n_B(x)$
  - additional parameters ('viscosities') for a non-ideal fluid
- Ideal ("inviscid") fluid $\equiv$ local thermal equilibrium $\Longrightarrow$ isotropy
- $T^{\mu\nu}_{(0)} = \mathrm{diag}\,(\epsilon,\, P,\, P,\, P)$ in the local rest frame at $x$: $u^\mu(x) = (1, 0)$
- After a boost to the laboratory frame, this becomes:

$$T^{\mu\nu}_{(0)} = (\epsilon + P)u^\mu u^\nu - P g^{\mu\nu}\,, \qquad J_B^\mu = n u_B^\mu$$

- $T^{\mu\nu}$: 4 equations, 5 unknown quantities: $\epsilon(x)$, $P(x)$, $v^i(x)$ $(i = 1, 2, 3)$
- Use the equation of state $\epsilon = \epsilon(P)$ to close the system

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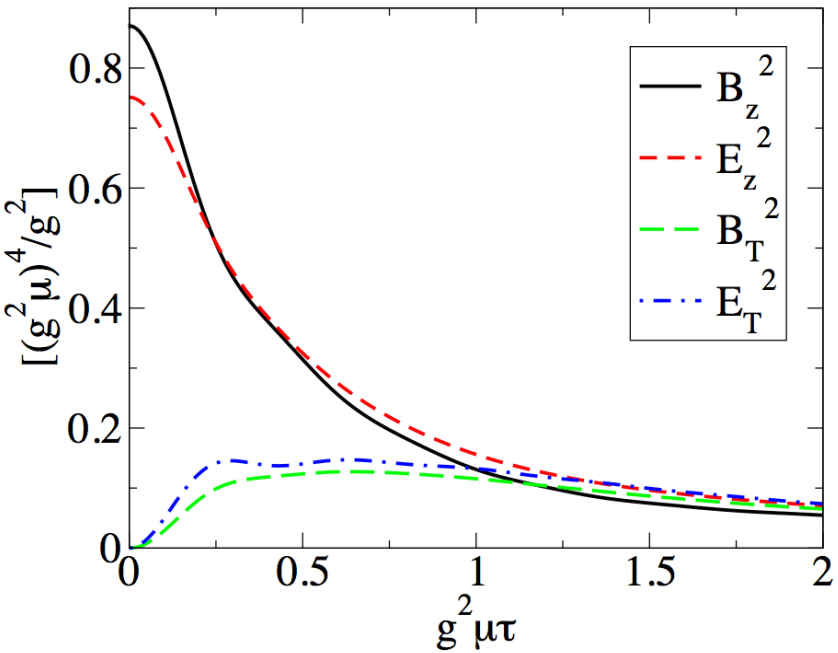
- E.g.: ideal gas of massless particles: $\epsilon = 3P$, hence $T^\mu_\mu = 0$

# Initial conditions & the thermalization puzzle

- One needs the initial conditions $\epsilon_0(\vec{x})$, $v_0^i(\vec{x})$ at some proper time $\tau_0$
- The isotropy seems inconsistent with the geometry of a heavy ion collision
  - strong dissymmetry between longitudinal and transverse directions




$$T^{\mu\nu}_{YM} = \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} - F^{\mu}_{\alpha} F^{\nu\alpha}$$

- $\tau = 0^+$: purely longitudinal fields

$$T^{\mu\nu}_{YM} = \frac{1}{2}(E_z^2 + B_z^2) \times \mathrm{diag}(1,1,1,-1)$$

- negative "longitudinal pressure"

- $\tau \sim 1/Q_s$: $T^{\mu\nu}_{YM} = \mathrm{diag}(\epsilon, \epsilon/2, \epsilon/2, 0)$ : zero "longitudinal pressure"
- "Rapid thermalization at some time $\tau_0 \lesssim 1$ fm" (?)

# Viscous hydrodynamics

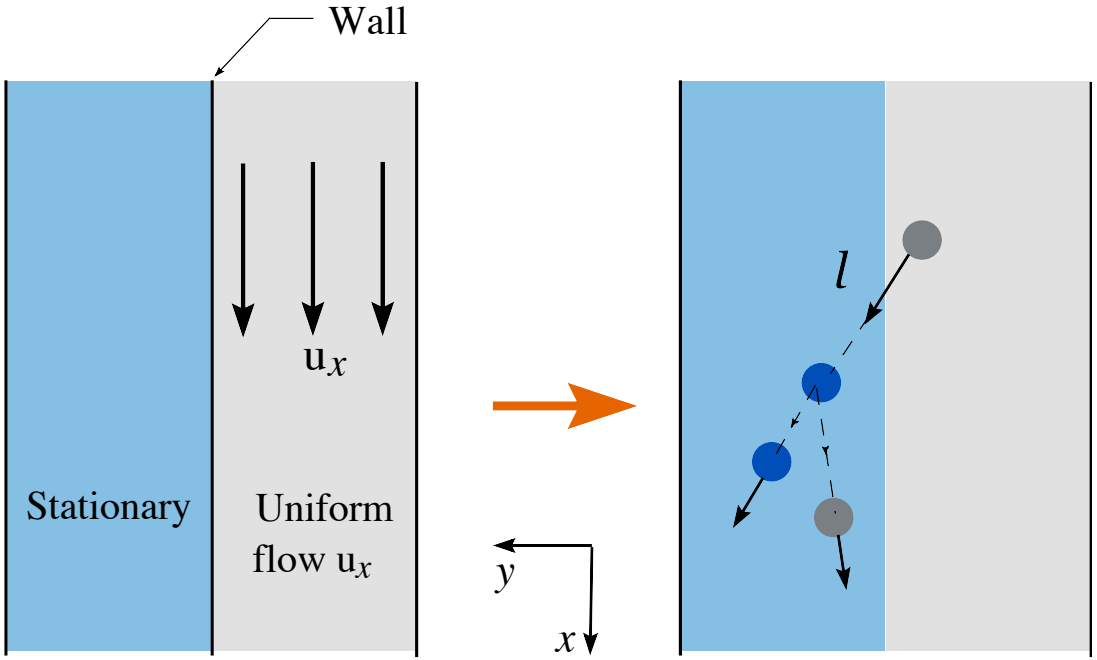
- Deviations from local thermal equilibrium are captured by viscous hydro
- Ideal hydro: zeroth order in the gradient expansion $\Longrightarrow$ no dissipation
- Dissipative effects are described by higher orders: $T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$
- The first-order terms: $\partial^\mu u^\nu$ or $g^{\mu\nu}(\partial \cdot u)$ are weighted by the viscosities
  - shear viscosity $\eta$, bulk viscosity $\zeta$: "transport coefficients"
  - measure the fluid ability to transfer energy and momentum along directions orthogonal to that of the flow
  - heuristically: friction between the different layers of the fluid
- Non-relativistic limit $v \ll 1 \Longrightarrow$ Navier-Stokes equation ($\rho =$ mass density)

$$\rho \left( \frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla} \right) \vec{v} = -\vec{\nabla} P + \eta \nabla^2 \vec{v} + \zeta \vec{\nabla} \left( \vec{\nabla} \cdot \vec{v} \right)$$

- First-order relativistic hydro is inconsistent with causality, but this can be fixed by adding second-order terms ("Müller-Israel-Stewart theory")

# Shear viscosity

- 2 layers of a same fluid, one flowing the other one at rest, separated by a wall
- Remove the wall: the two layers would not get mixed for ideal flow $(\eta = 0)$
- $\eta$ : a measure of a fluid ability to transfer $p_x$ in the $y$ direction




$$\frac{1}{A}\frac{\mathrm{d}p_x}{\mathrm{d}t} = -\eta\,\frac{\mathrm{d}u_x}{\mathrm{d}y}$$

$$\eta \sim \ell\rho v_{\rm th} \sim \ell\epsilon/\rho v_{\rm th}$$

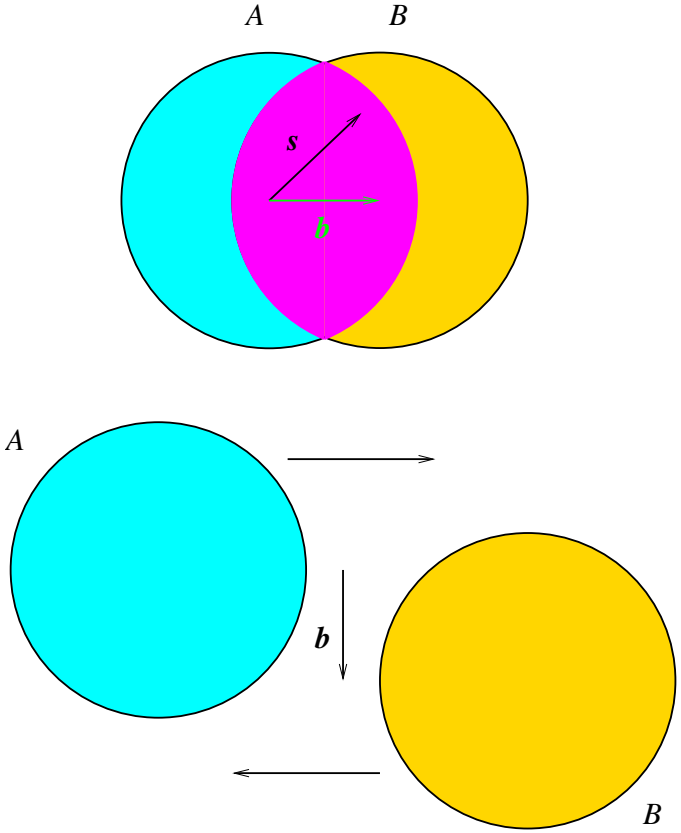
- $v_{\rm th}$ average molecular velocity ($\gg$ fluid velocity $v$)

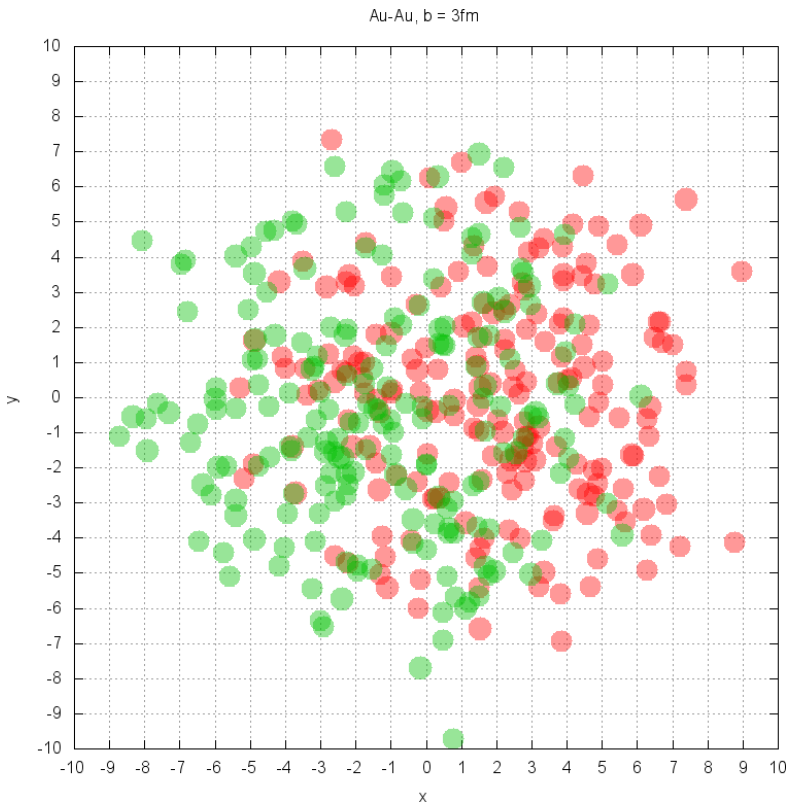
- Proportional to the mean free path: $\ell \propto 1/(n\sigma) \sim 1/\alpha_s^2$
  - larger for weakly coupled systems ! (Maxwell, 1860)

# Initial conditions: from smoothness to granularity

- Left: mean field description — a smooth distribution for the nuclear matter

Optical Glauber



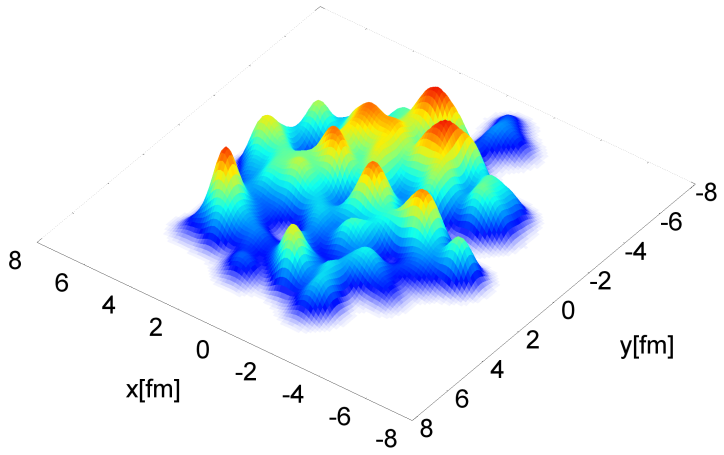




- Right: random distribution of the nucleons inside each nucleus

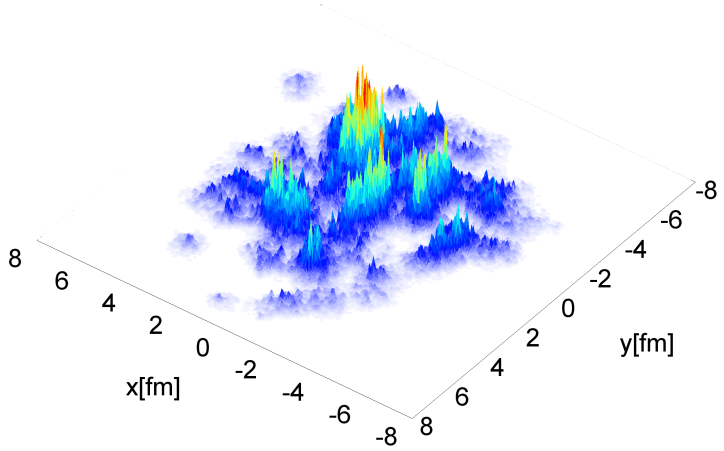
  $\Longrightarrow$ fluctuations (Monte-Carlo Glauber)

# Initial conditions: from nucleons to Glasma

- Left: random superposition of nucleons within the nuclear disks




- Monte-Carlo Glauber (nucleons)
- size of flucts: $R_p \sim 1$ fm

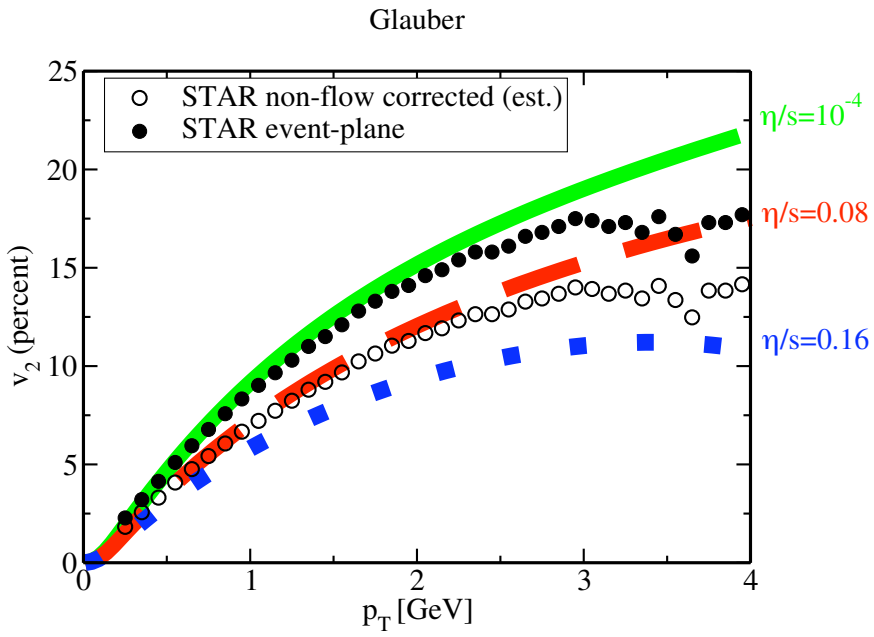


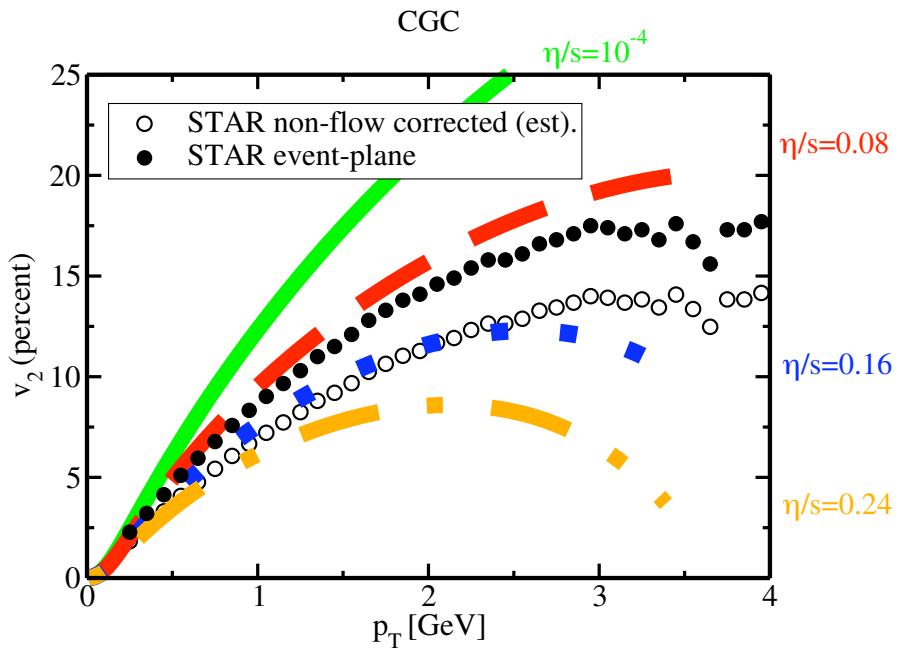

- Glasma (color flux tubes)
- size of flucts: $1/Q_s \sim 0.2$ fm

- Right: Glasma simulation (classical Yang–Mills equations with randomly distributed color charges) *(see review by Gelis and Schenke, 2016)*

# Hydro simulations for $v_2$ *(from Luzum and Romatschke, 08)*

- Early simulations used ideal hydro + Monte-Carlo Glauber (nucleons)
  - required early equilibration $\tau_0 \lesssim 1$ fm/c: very puzzling
- Glasma generates a larger initial eccentricity $\varepsilon_2$
  - this leaves the place for a larger viscosity, i.e. for deviations from local thermal equilibrium

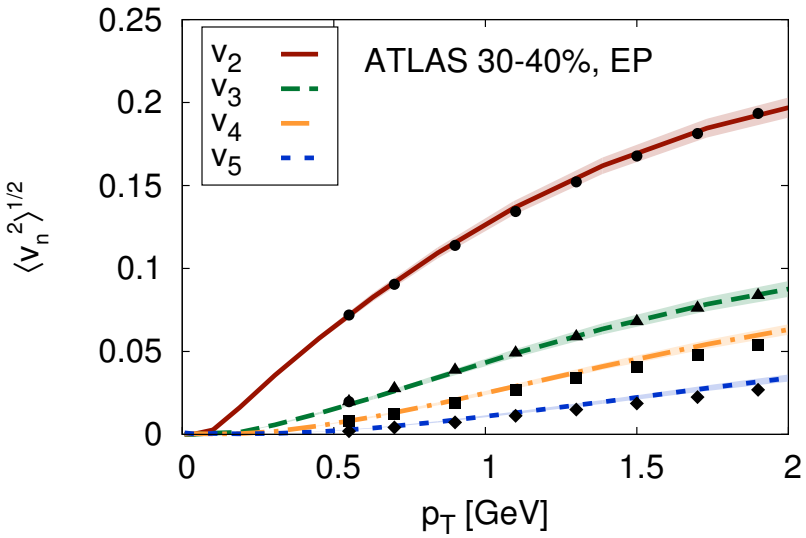


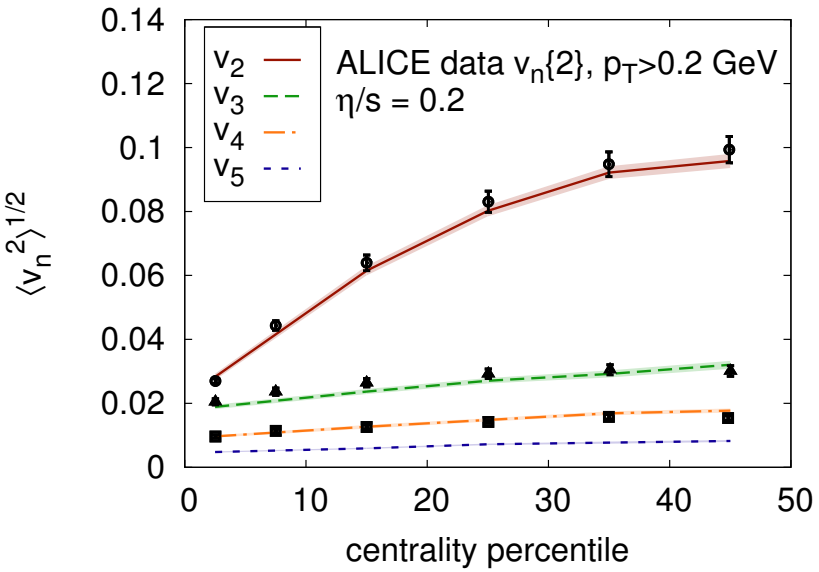




- The CGC fits prefer a larger value for the viscosity/entropy density ratio

# Hydro simulations for $v_2$ *(Gelis and Schenke, 2016)*

- Early simulations used ideal hydro + Monte-Carlo Glauber (nucleons)
  - required early equilibration $\tau_0 \lesssim 1$ fm/c: very puzzling
- Glasma generates a larger initial eccentricity $\varepsilon_2$
  - viscous hydro + Glasma yields very good fits to all $v_n$'s with $\eta/s = 0.2$




- This value $\eta/s$ seems still too small to be consistent with weak coupling

# Viscosity over entropy density ratio

- $\eta \sim \ell \times \epsilon$ ($\ell$ : mean free path; $\epsilon$ : energy density)
- Thermodynamics: $\epsilon + P = Ts \implies \epsilon/s \sim T \sim 1/\lambda_{\rm th}$
- Uncertainty principle $\implies$ ideal fluids ($\eta = 0$) cannot exist in nature

$$\frac{\eta}{s} \sim \frac{\ell}{\lambda_{\rm th}} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}} \gtrsim \hbar$$

- Weakly coupled QGP *(Arnold, Moore, Yaffe, 2003)* :

$$\frac{\eta}{s} \sim \frac{\hbar}{\alpha_s^2 \ln(1/\alpha_s)} \gg \hbar$$

- Conjectured limit at strong coupling *(Kovtun, Son, Starinets, 05)*

$$\frac{\eta}{s} \to \frac{\hbar}{4\pi} \quad \text{when} \quad \lambda \equiv g^2 N_c \to \infty \quad \text{(AdS/CFT)}$$

- The value preferred by the fits is at most a few times $1/4\pi \simeq 0.08$

# RHIC serves up the perfect liquid !

## RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted -- raising many new questions

Monday, April 18, 2005

TAMPA, FL -- The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) -- a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory -- say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

- 'Strongly–coupled quark–gluon plasma' or 'perfect fluid'
- Applications of the AdS/CFT correspondence to heavy ion collisions
- Two main arguments in favor of strong coupling:
  - small $\eta/s$ ratio from the hydro fits to the flow coefficients
  - small thermalization time needed for consistency of hydro

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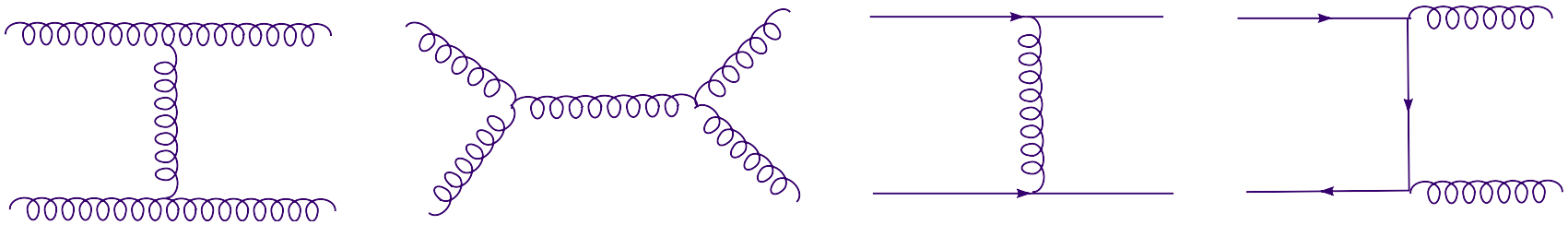
- 'Strongly–coupled quark–gluon plasma' or 'perfect fluid'
- Applications of the AdS/CFT correspondence to heavy ion collisions
- Two main arguments in favor of strong coupling:
  - small $\eta/s$ ratio from the hydro fits to the flow coefficients
  - small thermalization time needed for consistency of hydro
- These 2 conditions can also be satisfied, or circumvented, at weak coupling

# Mean free path at weak coupling (1)

- A weakly coupled QGP in thermal equilibrium with temperature $T$
- Thermal (quasi)particles: quarks and gluons with energy and momenta of order $T$, whose properties are slightly dressed by the interactions
- Number density $n \sim T^3$, energy density $\epsilon \sim T^4$
- Mean free path: typical distance/time between two successive collisions

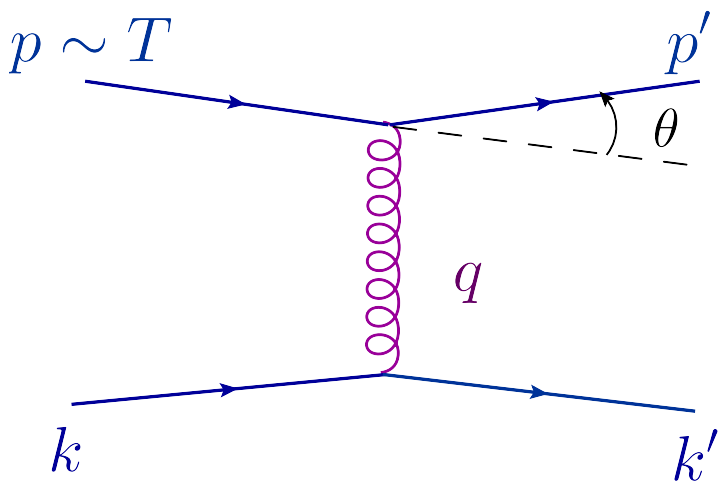
$$\ell \sim \frac{1}{\Gamma}\,, \qquad \Gamma = \text{ collision rate } = n\sigma v_{rel} \sim T^3\sigma$$

- $\sigma$ : cross-section for binary collisions
- Weak coupling $\Longrightarrow$ the dominant mechanism is $2 \to 2$ elastic scattering



# Mean free path at weak coupling (2)

- Coulomb scattering (one gluon exchange), deflection angle $\theta \simeq q_\perp/p$




$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp^2} \sim \frac{\alpha_s^2}{q_\perp^4}$$

- the total cross-section

$$\sigma = \int \mathrm{d}q_\perp^2 \, \frac{\alpha_s^2}{q_\perp^4}$$

- ... is infrared ($q_\perp \to 0$) divergent

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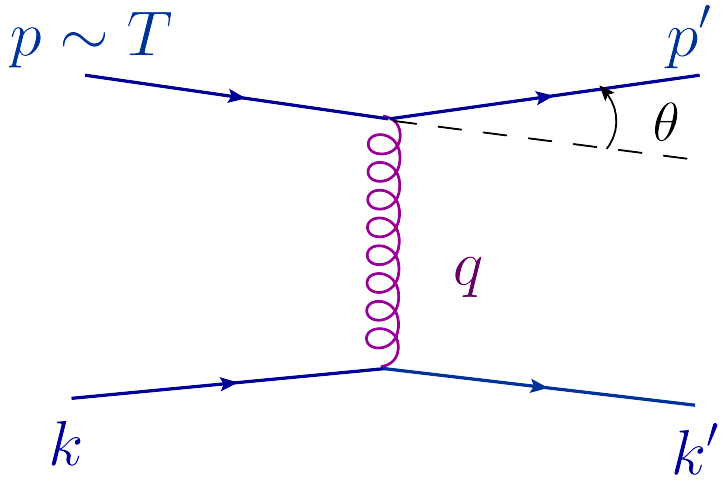



$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_\perp^2} \sim \frac{\alpha_s^2}{q_\perp^4}$$

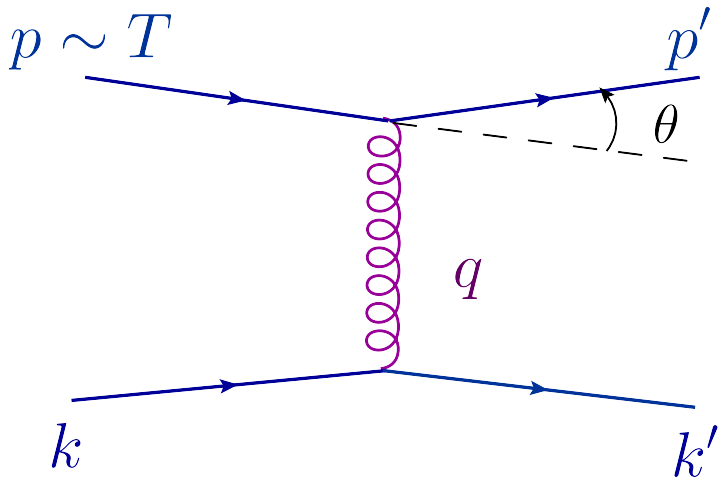
- the total cross-section

$$\sigma = \int_{\alpha_s T^2} \mathrm{d}q_\perp^2 \, \frac{\alpha_s^2}{q_\perp^4} \simeq \frac{\alpha_s}{T^2}$$

- In the plasma, color interactions are screened over a distance $\lambda_D \sim 1/gT$, that is, at the momentum scale $q_\perp^2 \sim \alpha_s T^2$ (Debye screening)
- $\ell_c \sim 1/(\alpha_s T)$ = m.f.p. for small-angle scattering: $\theta \sim gT/T \sim g$
  - also for color relaxation: even soft collisions can change the color state
- Not the relevant m.f.p. for processes which require momentum transfer, like viscosity: large-angle scattering

# Mean free path at weak coupling (3)

- The transport cross-section: weighted by $1-\cos\theta \simeq \theta^2/2$ with $\theta \simeq q_\perp/p$




$$\sigma_{tr} = \int \mathrm{d}q_\perp^2 \, \frac{q_\perp^2}{p^2} \, \frac{\alpha_s^2}{q_\perp^4}$$

$$\sigma_{tr} = \frac{\alpha_s^2}{T^2} \int_{\alpha_s T^2}^{T^2} \frac{\mathrm{d}q_\perp^2}{q_\perp^2} = \frac{\alpha_s^2}{T^2} \ln \frac{1}{\alpha_s}$$

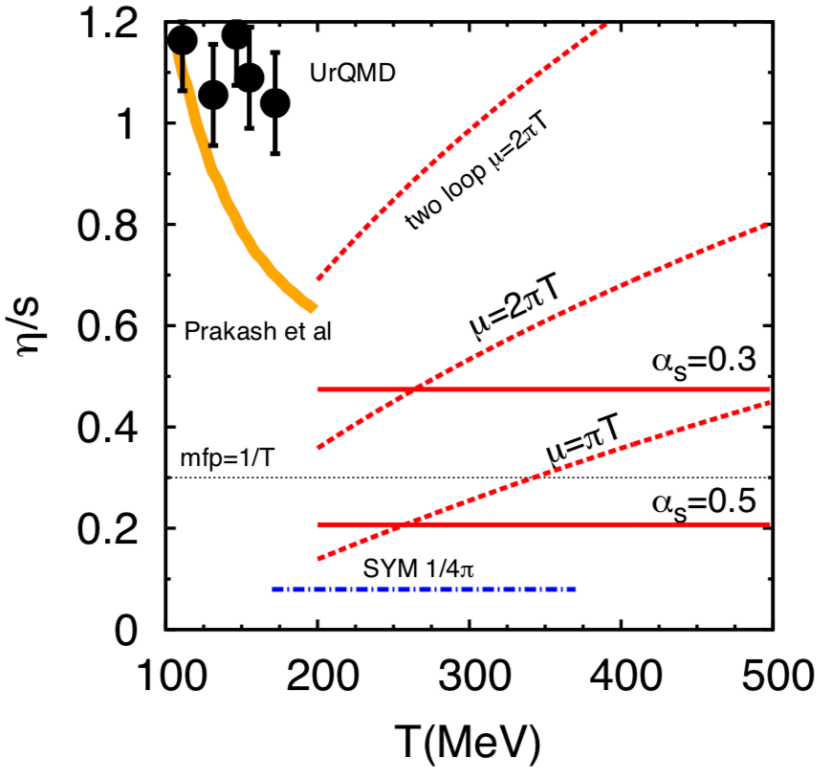
- Logarithmically sensitive to all "semi-hard" scales $gT \ll q_\perp \ll T$

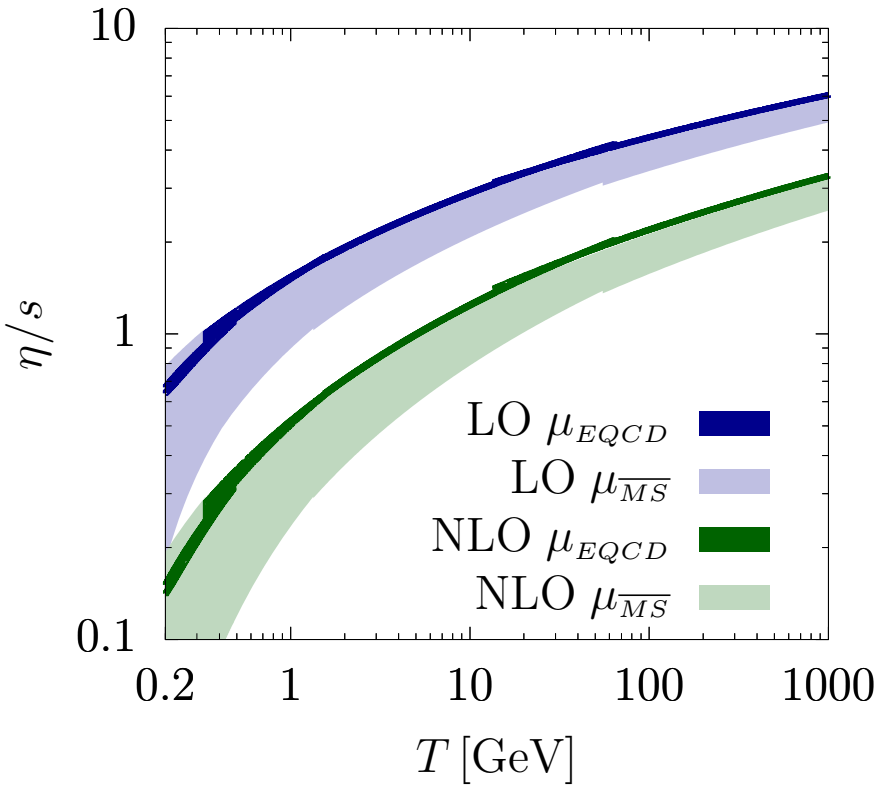
$$\ell \sim \frac{1}{\sigma_{tr} T^3} \sim \frac{1}{\alpha_s^2 T \ln(1/\alpha_s)} = \text{m.f.p. for large-angle scattering}$$

- $\ell \gg \ell_c$: changing momentum is more difficult than changing the color
- Shear viscosity at weak coupling: $\eta \sim \ell\epsilon \sim \frac{T^3}{\alpha_s^2 \ln(1/\alpha_s)}$

# $\eta/s$ in pQCD: from LO to NLO

- Left: a compilation of the leading-order result *(Arnold, Moore, Yaffe, 2003)* by D. Teaney, arXiv:0905.2433
  - fixed coupling & running coupling with 1-loop and 1-loop beta function




- Right: the recent NLO calculation vs to the LO result (2-loop running) Ghiglieri, Moore and Teaney, arXiv:1802:09535 (based on kinetic theory)

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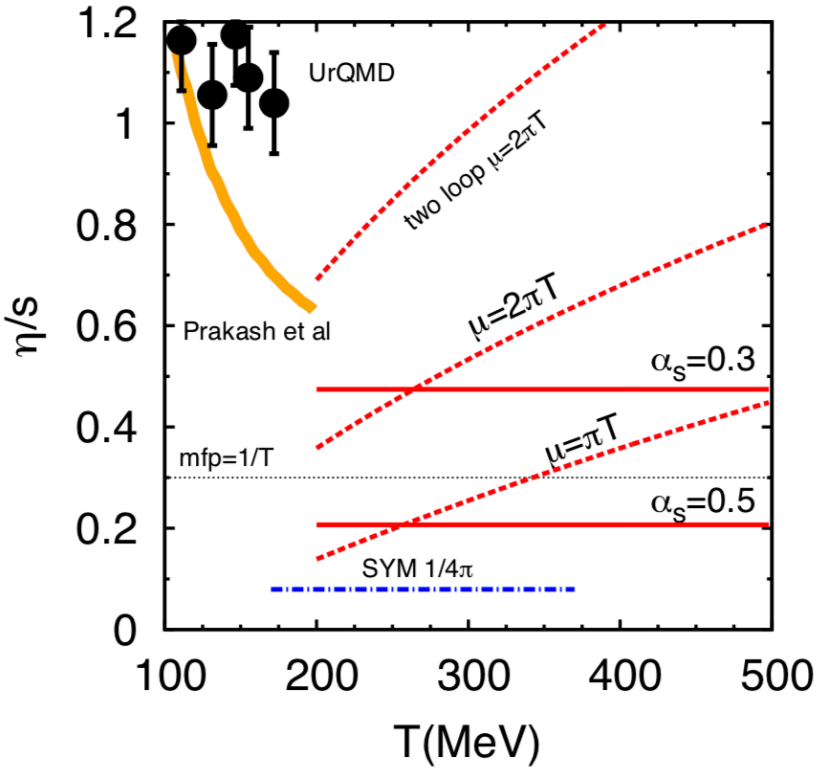
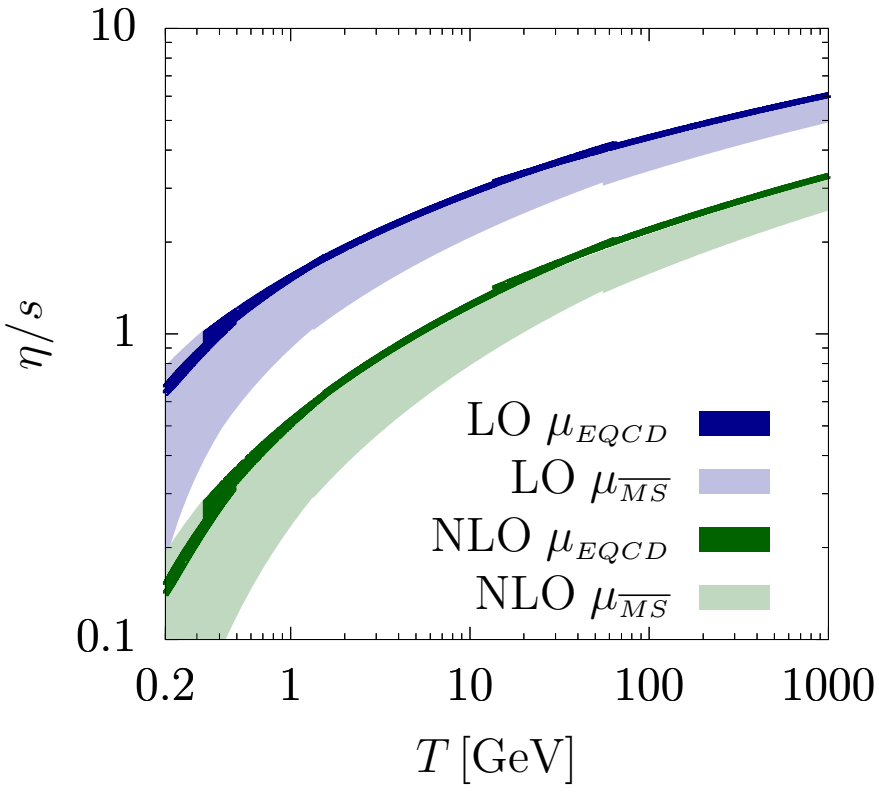



- Right: the recent NLO calculation vs to the LO result (2-loop running)
  - at $T \sim 300$ MeV, the result is reduced by a factor of 4: $\eta/s \sim 0.2$

# Kinetic theory in a nut shell

- For weakly coupled systems, kinetic theory can be used to describe the non-equilibrium evolution at intermediate times $1/Q_s \lesssim \tau \lesssim \tau_0$
- Follow the non-equilibrium evolution of the phase-space occupation numbers

$$f_s(t,\mathbf{x},\mathbf{p}) \equiv \frac{(2\pi)^3}{\nu_s}\frac{\mathrm{d}N_s}{\mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{p}}$$

  - $s = g,\, q,\, \bar{q}\ldots;\ \nu_g = 2(N_c^2-1),\quad \nu_q = \nu_{\bar{q}} = 2N_c$

- Meaningful when $f_s \ll 1/\alpha$ (which includes $f \gg 1$ at weak coupling)
- It smoothly matches onto classical Yang-Mills dynamics when $f \gg 1$ and onto hydro when off-equilibrium deviations are small: $\delta f \ll f_{\rm eq}$
- All the observables can be computed from $f_s(t,\mathbf{x},\mathbf{p})$

$$T^{\mu\nu}(x) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{p^\mu p^\nu}{E_p}\sum_s \nu_s f_s(t,\mathbf{x},\mathbf{p})$$

# Boltzmann equation

- Two types of interactions: external force and collisions among the particles

$$\left(\frac{\partial}{\partial t} + \mathbf{v_p} \cdot \frac{\partial}{\partial \mathbf{x}} + \mathbf{F}_{\rm ext} \cdot \frac{\partial}{\partial \mathbf{p}}\right) f(t, \mathbf{x}, \mathbf{p}) = -C[f]$$

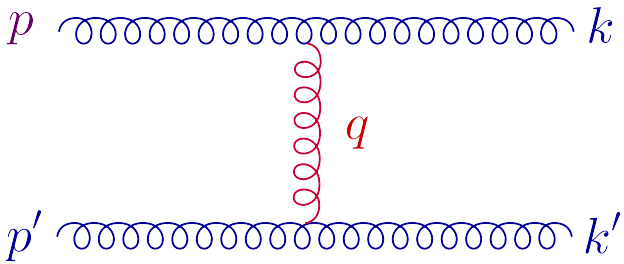
- $C[f]$: "collision term": non-linear in $f$, non-local in $\mathbf{p}$, but local in $\mathbf{x}$
- Variation time scales much larger than the duration of the collision process
  - e.g. $\Delta t \gg 1/(gT)$ for $2 \to 2$ collisions near equilibrium
- $C[f_{\rm eq}]$=0: collision term vanishes in thermal equilibrium $f_s(t, \mathbf{x}, \mathbf{p}) \to n_s(p)$

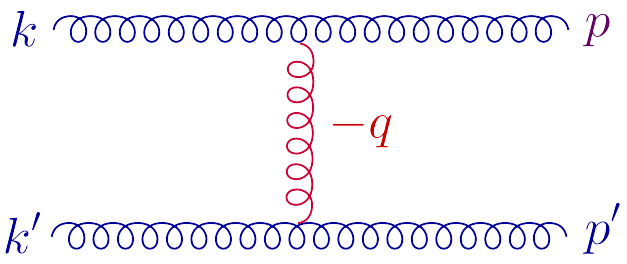
$$n_g(p) = \frac{1}{{\rm e}^{\beta p} - 1}\,, \qquad n_{q,\bar{q}}(p) = \frac{1}{{\rm e}^{\beta(E_p \mp \mu)} + 1}$$

  - Bose-Einstein for gluons, Fermi-Dirac for quarks
- Collisions can be either elastic (e.g. $2 \to 2$) or inelastic (e.g. $2 \to 3$)

# Elastic collisions

- $2 \to 2$ collisions: generally argued to be the LO piece at weak coupling




- 'Gain' and 'loss' terms: the gluon $\mathbf{p}$ is the one that is measured

$$\mathcal{C}_{\rm el}[f] \,=\, \int_{\mathbf{p}',\mathbf{k},\mathbf{k}'} \frac{|\mathcal{M}|^2}{(2p)(2p')(2k)(2k')}\,\Phi[f]$$

$$\Phi[f] = f_{\mathbf{p}} f_{\mathbf{p}'}[1+f_{\mathbf{k}}][1+f_{\mathbf{k}'}] - f_{\mathbf{k}} f_{\mathbf{k}'}[1+f_{\mathbf{p}}][1+f_{\mathbf{p}'}]$$

- 5 conserved quantities: particle number and the 4–momentum
- It vanishes at (local) thermal equilibrium: $\mathcal{C}_{\rm el}[n] = 0$ ("detailed balance")

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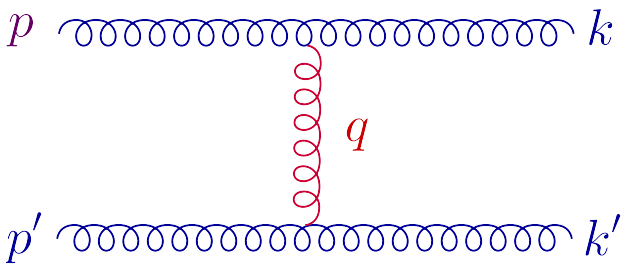


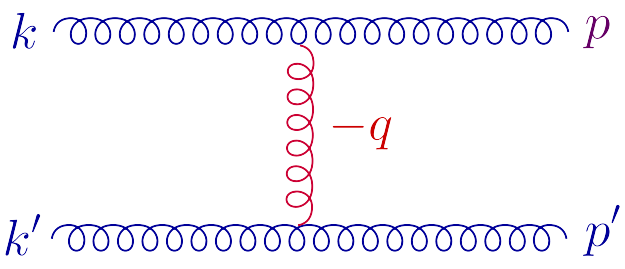


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- It also vanishes in the limit of zero momentum transfer $q = |\mathbf{q}| \to 0$
  - transport cross-section: weighted by a factor $q^2$ at small $q$

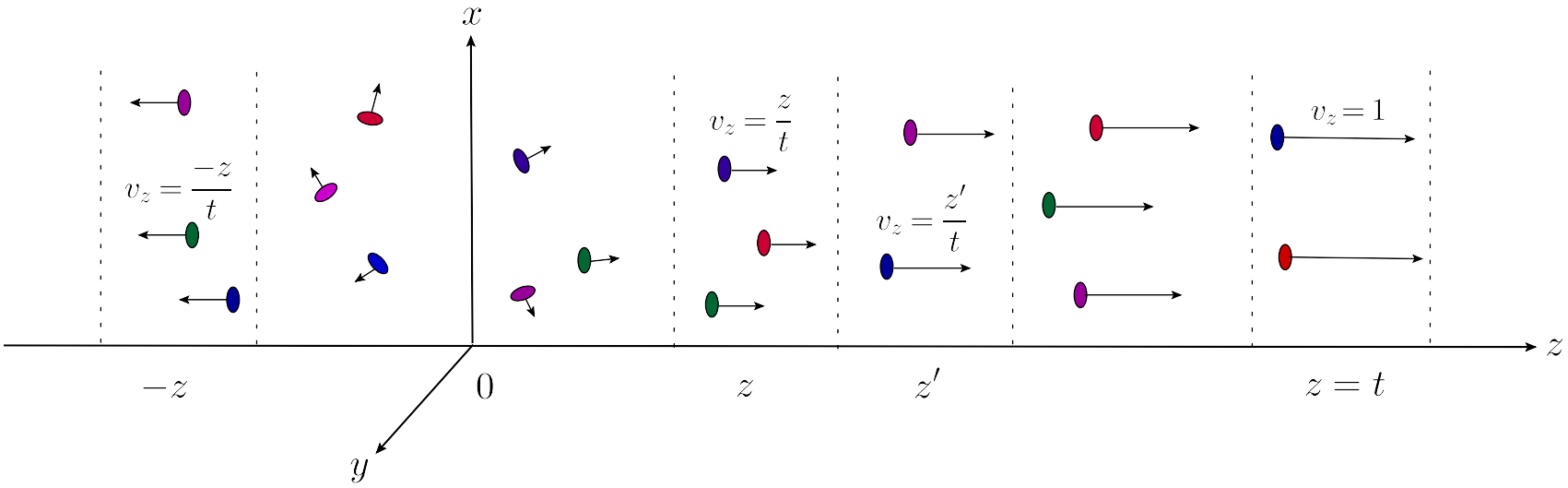
# The thermalization puzzle

- An off-equilibrium perturbation typically dies away over a time $\tau_{\rm rel} \sim \ell/v_{\rm th}$
- At weak coupling, $\tau_{\rm rel} \sim \ell$ is parametrically large (in units of $\lambda_{\rm th} = 1/T$)

$$\frac{\tau_{\rm rel}}{\lambda_{\rm th}} \sim \frac{\eta}{s} \sim \frac{1}{\alpha_s^2 \ln(1/\alpha_s)} \gg 1$$

- The folklore says: "Fast thermalization is not possible at weak coupling"
- Not quite true: recall the NLO calculation of $\eta/s$ (Ghiglieri et al, arXiv:1802:09535)
  - at $T \sim 300$ MeV, the result is reduced by a factor of 4: $\eta/s \sim 0.2$
- The actual problem is the rapid longitudinal expansion
  - the partonic system becomes more and more dilute
  - collisions occur less often and have little chances to equilibrate
- A solution to that is provided by radiation ("inelastic collisions")

# Anisotropy from the longitudinal expansion

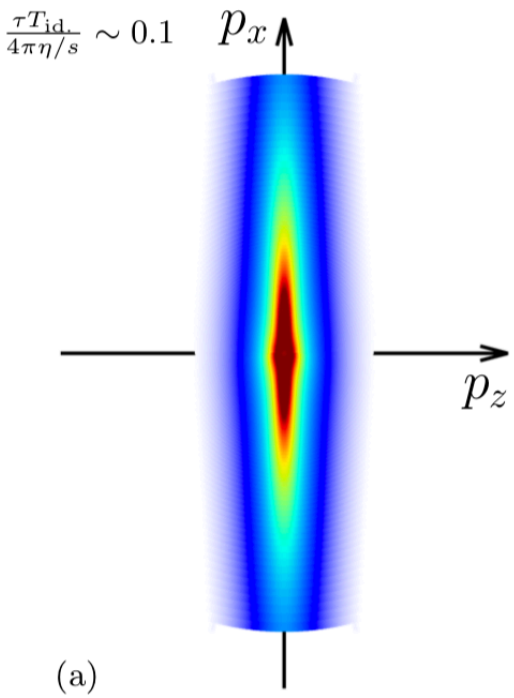



- At time $t \sim 1/Q_s$, gluons are freed from the nuclei with $p_\perp \sim Q_s$ ("hard") and occupation numbers $f_0 \sim 1/\bar{\alpha}$
- At later times $t \gg 1/Q_s$, all the gluons located at $z$ have $v_z = z/t$
  - the distribution in velocity (or momentum) is highly anisotropic
- Focus on midrapidity for definiteness: $\eta \sim 0$ (say, $z \sim 1/Q_s$)
  - physics is boost invariant: replace $t \to \tau \equiv \sqrt{t^2 - z^2}$ for any other $\eta$

$$v_z \simeq \frac{p_z}{p_\perp} \sim \frac{1}{Q_s \tau} \implies p_z \simeq \frac{1}{\tau} \ll p_\perp \sim Q_s \quad \text{when} \quad Q_s \tau \gg 1$$

# The "Bottom-up" scenario *(Baier, Mueller, Schiff, Son '01)*

- This would be the distribution in the absence of collisions
- Elastic $2 \to 2$ scattering competes with the longitudinal expansion
  - $p_z$ decreases less fast with $\tau$: $p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}$




- numerical calculation by Kurkela et al, arXiv:1805.00961
- gluon distribution in the $(p_x, p_z)$ plane as a function of the scaled time (with $\eta/s = 0.62$)

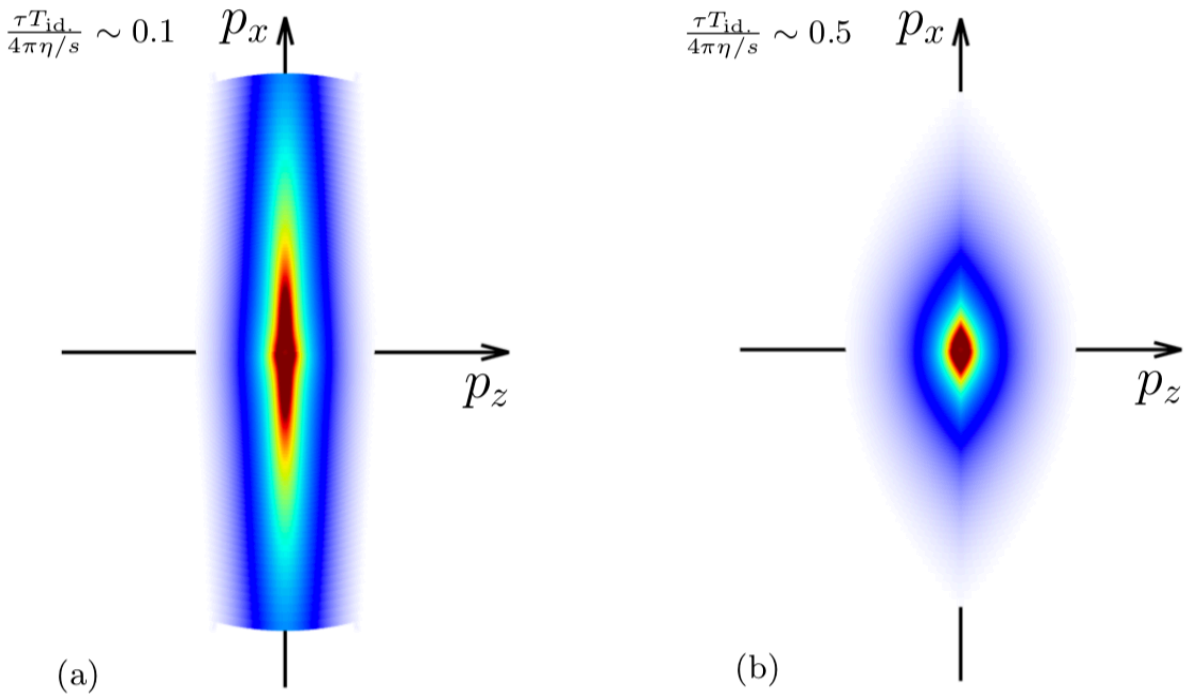
$$\frac{\tau}{\tau_{\rm rel}(\tau)} \equiv \frac{\tau T(\tau)}{\eta/s}$$

- the color scale: the occupation number in the range $0.1 < f < 5$

- Anisotropy less pronounced than for free streaming, but steadily increasing

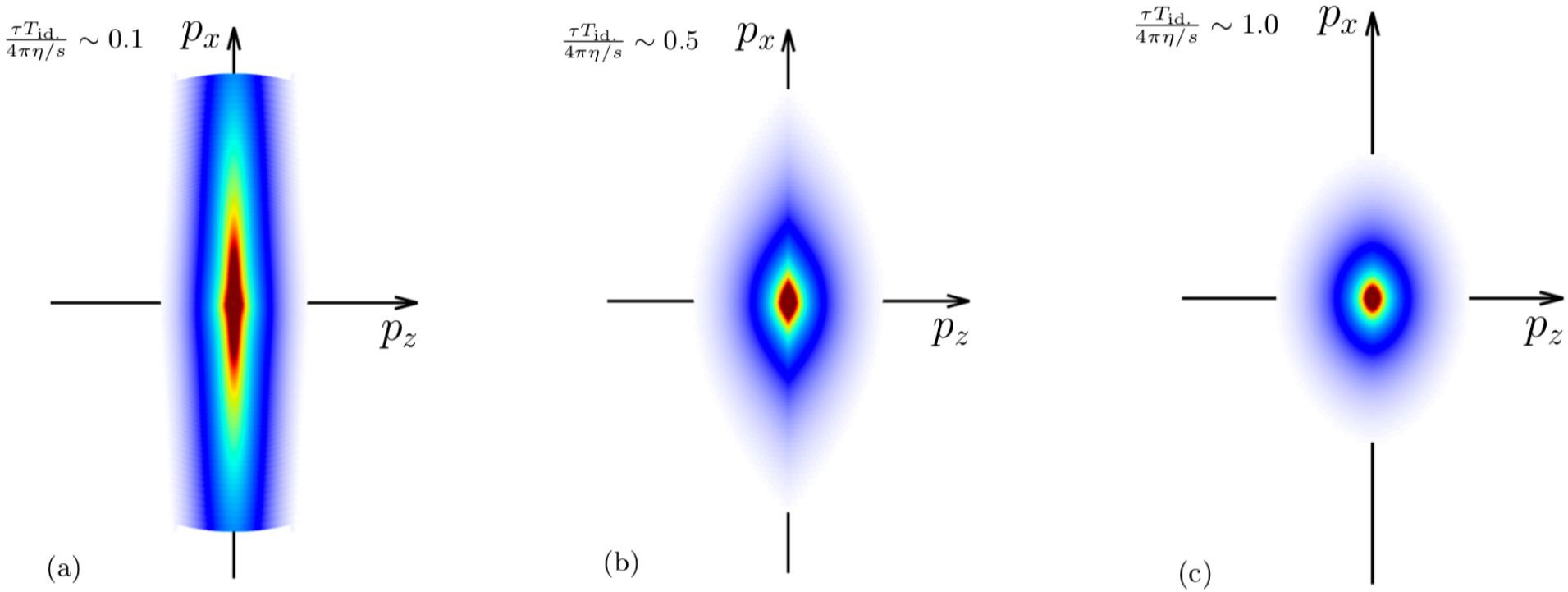
# The "Bottom-up" scenario *(Baier, Mueller, Schiff, Son '01)*

- The "hard" ($p_\perp \sim Q_s$) gluons can also suffer "inelastic collisions"
  - collisions which trigger emissions of softer gluons
- The soft gluons are emitted isotropically
  - they escape the problem of the longitudinal expansion
- They can thermalize, over a time $\sim \tau_{\rm rel}$, via elastic $2 \to 2$ scattering



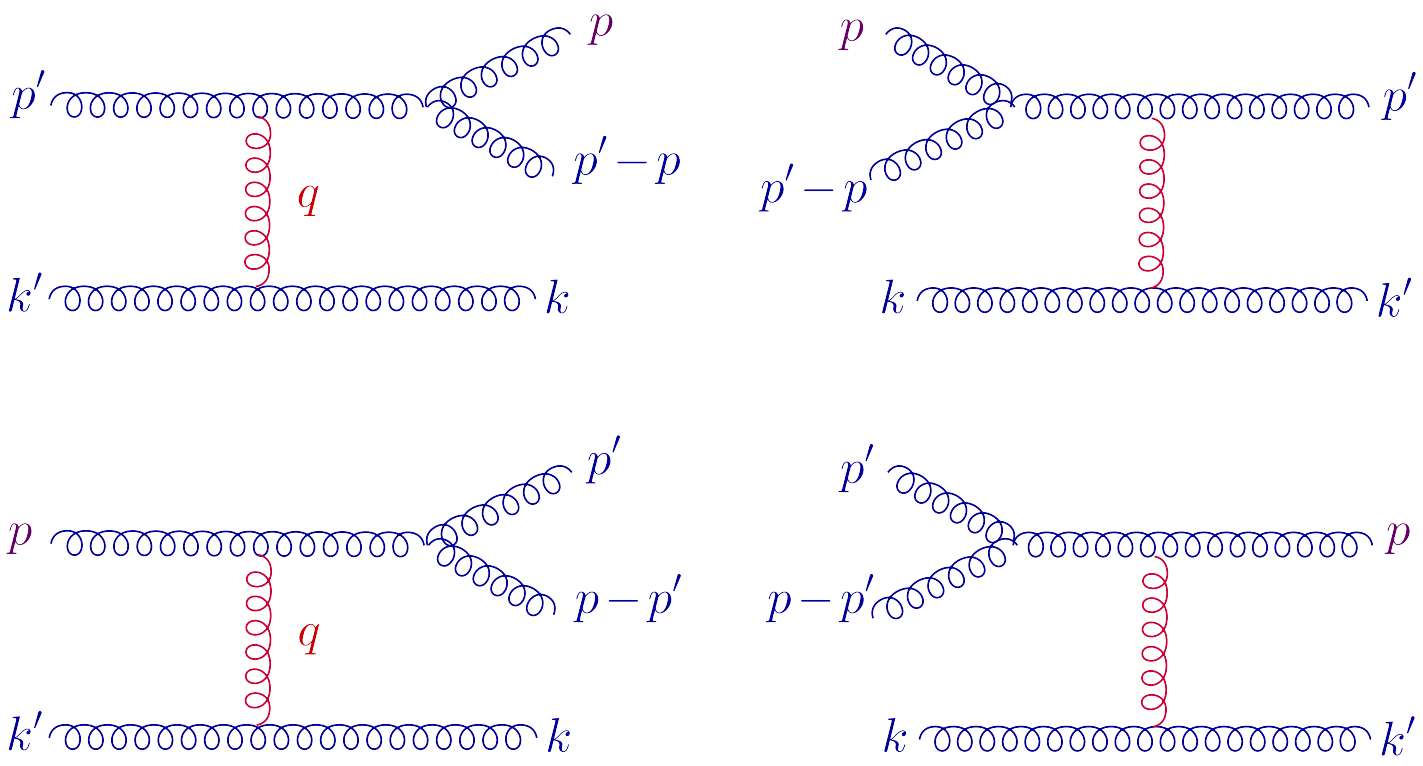
# The "Bottom-up" scenario *(Baier, Mueller, Schiff, Son '01)*

- The "hard" ($p_\perp \sim Q_s$) gluons can also suffer "inelastic collisions"
  - collisions which trigger emissions of softer gluons
- The hard particles scatter off the soft thermal both
- They efficiently lose energy via medium-induced radiation
- The distribution approaches local thermal equilibrium



# Medium-induced branchings

- The prototype: $2 \leftrightarrow 3$ inelastic scattering (single scattering)

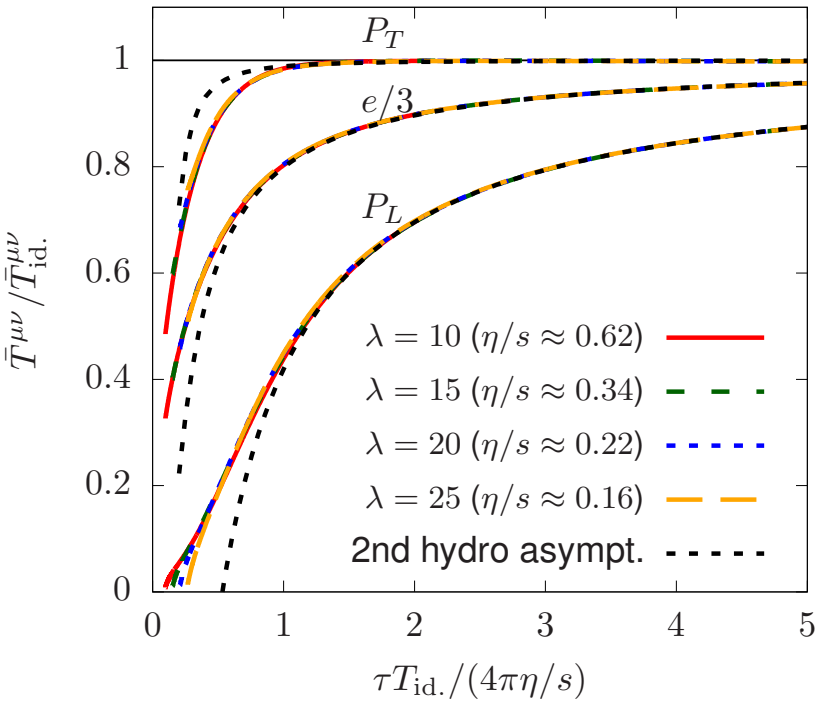



- Both splittings $(2 \to 3)$ and recombinations $(3 \to 2)$
- The splittings (recombinations) can contribute both to "gain" and to "loss"
- Actually, multiple scattering is important ("LPM effect)

# Hydrodynamisation in the "Bottom-up" scenario

*(Kurkela and Zhu, arXiv:1506.06647; Kurkela et al, arXiv:1805.00961)*

- Numerical solutions to the Boltzmann equation with $\mathcal{C}[f] = \mathcal{C}_{\rm el}[f] + \mathcal{C}_{\rm inel}[f]$
- Highly anisotropic initial conditions: $P_L \ll P_T$




- different values of the coupling $(\eta/s)$

$$\lambda \equiv g^2 N_c = 4\pi\alpha_s N_c$$

- viscous hydro (2nd order in gradients)
- hydro: universal curve of the rescaled time

$$\frac{\tau}{\tau_{\rm rel}(\tau)} \equiv \frac{\tau T(\tau)}{\eta/s}$$

- Kinetic theory $\Rightarrow$ universal curve quasi-independent of the coupling (or $\eta/s$)
- Perfect matching with viscous hydro for $\tau \gtrsim \tau_{\rm rel}$
- ... albeit the anisotropy is still pronounced: $(P_T - P_L)/P_T \sim 30\%$

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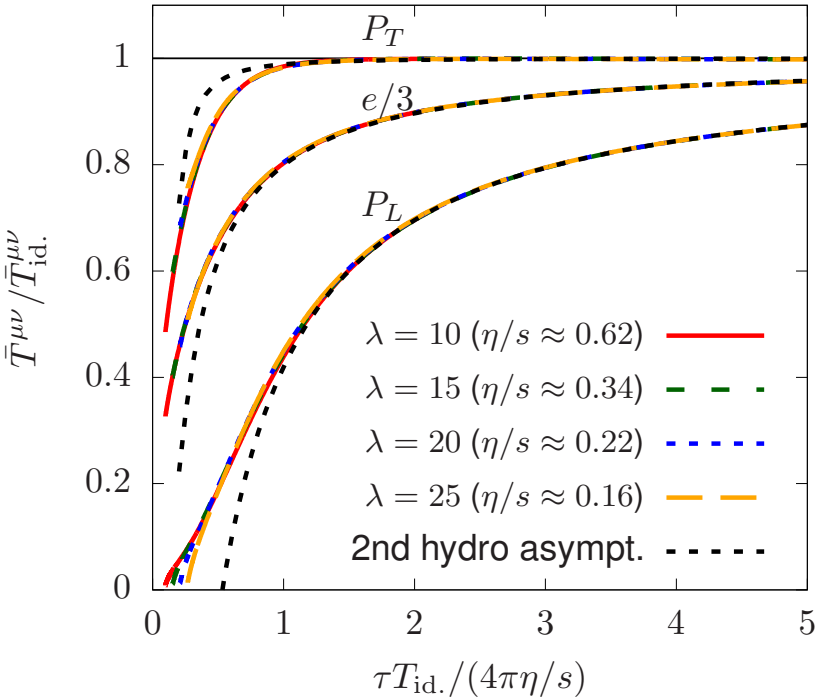


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- The fact that the data for the flow coefficients $v_n$ can be well described by viscous hydro does not mean that the QGP has reached local equilibrium !

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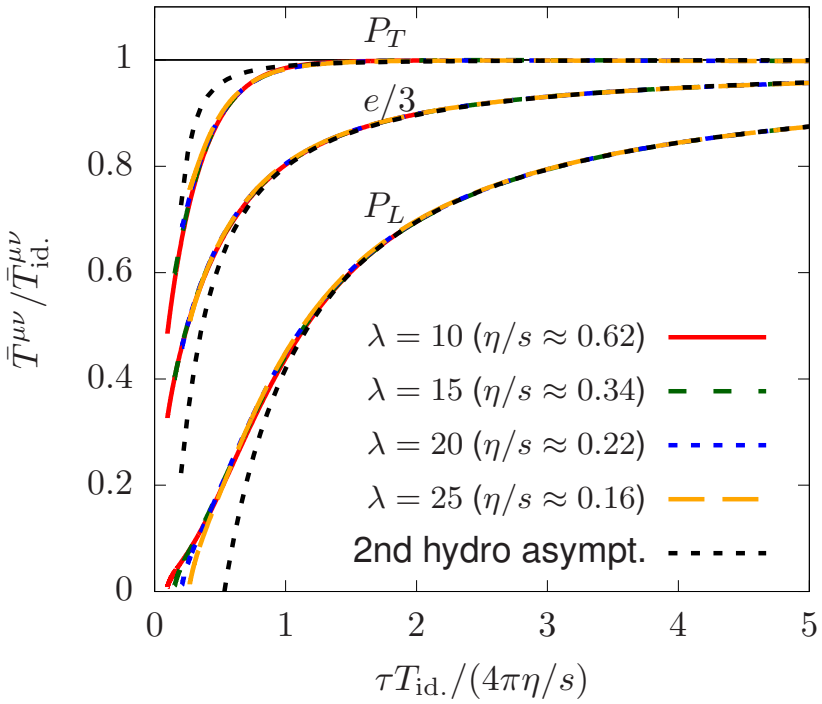


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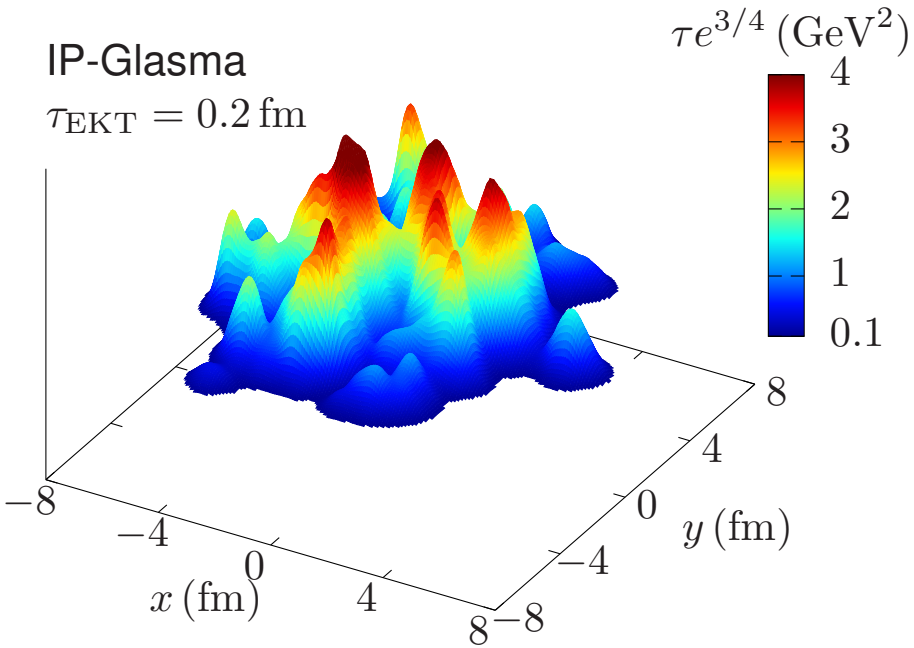
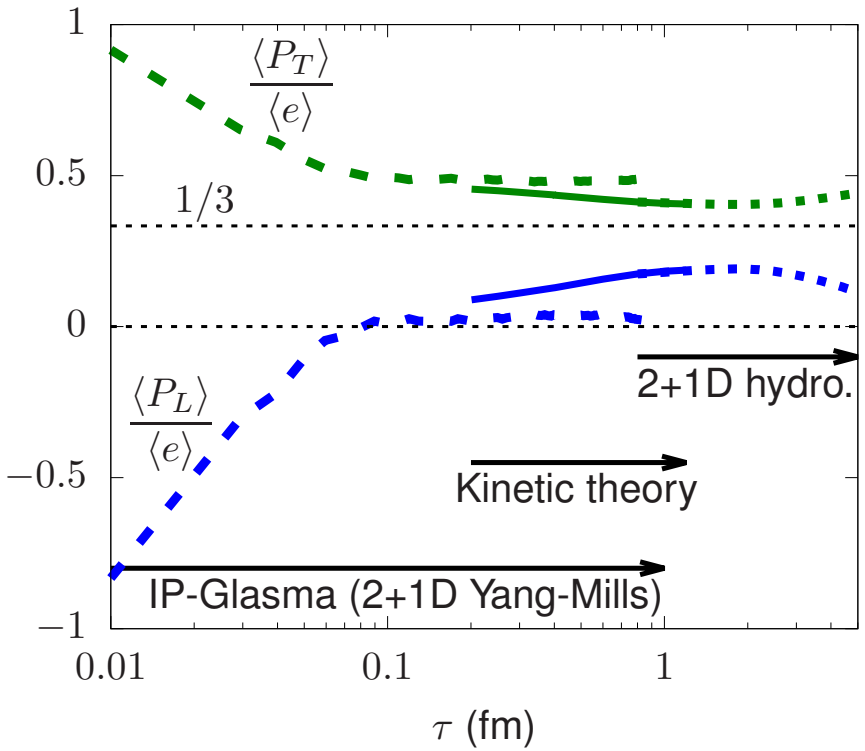
- In proper units, the results are independent of the coupling
- In fact, the same conclusion (hydrodynamisation w/o thermalization) has been reached at infinite coupling, within AdS/CFT *(Chesler and Yaffe, 2008)*

# A sequence of effective theories *(Kurkela et al, 2018)*

- A more realistic simulation of a heavy ion collision
  - initial conditions at $\tau = 0_-$: McLerran-Venugopalan model
  - numerical solutions to classical Yang-Mills equations (glasma)
  - negative longitudinal pressure at times $0_+ < \tau < 1/Q_s = 0.1$ fm
  - matching onto kinetic theory at $\tau = 0.2$ fm
  - matching onto viscous hydro at $\tau = 2$ fm



# THANK YOU FOR YOUR ATTENTION !



... and have a good trip back home!