Introduction to perturbative QCD and factorization

Part 2: $e^+e^- \rightarrow \text{hadrons}$

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$e^+ e^- \rightarrow \text{hadrons}$

\[
R = \frac{\sigma(e^+ e^- \rightarrow X)}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)}
\]

for $\sqrt{s} \gg \text{resonance masses}$

- removing electroweak part $\sim \sum_X |A(\gamma^* \text{ or } Z^* \rightarrow X)|^2$
- among simplest applications of perturbative QCD
  - fully inclusive final state
  - no hadrons in initial state
- closely related theory description for

\[
R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + X)}{\Gamma(\tau \rightarrow \nu_\tau + e\nu_e)} \sim \sum_X |A(W^* \rightarrow X)|^2
\]
at lowest order in $\alpha_s$:

$$R_0 = N_c \sum_q e_q^2$$

from $\gamma^* \rightarrow q\bar{q}$ with $m_q = 0$

- expansion known up to $R = R_0 \left[ 1 + \frac{1}{\pi} \alpha_s + C_2 \alpha_s^2 + C_3 \alpha_s^3 + C_4 \alpha_s^4 \right]$
  - quark mass corrections also partly known
  - same for $\tau$ decays
  - suitable observables for $\alpha_s$ determination

- underlying concept: parton-hadron duality:

$$\sum_{X \in \text{partons}} |A(\gamma^* \rightarrow X)|^2 = \sum_{X \in \text{hadrons}} |A(\gamma^* \rightarrow X)|^2$$

- $\gamma^* \rightarrow$ partons valid description for short space-time $\sim 1/\sqrt{s}$
- subsequent dynamics changes final state, but not inclusive rate
A closer look at the $\mathcal{O}(\alpha_s)$ corrections

- expand $\mathcal{A}(q\bar{q}g) = gA_1 + \ldots$ and $\mathcal{A}(q\bar{q}) = A_0 + g^2A_2 + \ldots$

![Diagram](image)

- real corrections: extra partons in final state
- virtual corrections: loops in $\mathcal{A}$ or $\mathcal{A}^*$

- virtual corrections have UV divergences
  $\rightarrow$ standard renormalisation procedure

- real and virtual corrections: soft and collinear divergences
  - regions where gluon momentum $\rightarrow 0$ or $\propto$ momentum of $q$ or $\bar{q}$
  - cancel in sum over all graphs
A closer look at soft and collinear divergences

$$e^+ e^- \rightarrow \text{hadrons}$$

$A_1 A_0 A_1^* A_2^*$

- more detail $\sim$ blackboard
A closer look at soft and collinear divergences

- have soft (= IR) div. because of massless gluons
  same phenomenon in QED: soft photons $\rightarrow$ “IR catastrophe”

- have collinear (= mass) div. if set quark masses to zero
  could formally keep $m_q \neq 0$, but perturbative results not trustworthy if virtualities $\sim \text{MeV}^2$

- divergences cancel, result dominated by large virtualities
  otherwise could not use parton-hadron duality
A footprint of divergence cancellation: large logarithms

- Both soft and collinear divergences are logarithmic: \[ \int \frac{dE}{E} \int \frac{d\theta}{\theta} \]
- Fixing final-state momenta restricts integration region in real corrections, but not in virtual ones
  - For each emission get double logarithm \( \propto \alpha_s \log^2(\ldots) \) “Sudakov logarithms”
  - If logarithms are large must sum them to all orders in \( \alpha_s \) “resummation”
  - Can be done analytically for certain cases
  - Done by “parton showers” in Monte Carlo generators
Beyond inclusive final states: hadronic jets

- jet = “bunch of hadrons moving approx. in same direction”
- perhaps the most direct manifestation of quarks or gluons

\[ Q^2 = 25030 \text{ GeV}^2, \quad y = 0.56, \quad M = 211 \text{ GeV} \]
Beyond inclusive final states: hadronic jets

- jet = “bunch of hadrons moving approx. in same direction”
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Beyond inclusive final states: hadronic jets

- extend idea of parton-hadron duality: dynamics leading from partons (times $\sim 1/Q$) to final-state hadrons (times $\rightarrow \infty$) approx. conserves momentum (hadronisation effects $\sim \text{GeV}$)

- to minimise theory uncertainties:
  - define hadronic jets using an algorithm that is not sensitive to collinear and soft radiation (beyond perturbative control)

  “collinear and infrared safe jet algorithm”

- apply to partons in computation, to hadrons in measurement
- hadronisation corrections should then be moderate and typically decrease with jet $p_T$

estimate using Monte Carlo generators $\rightarrow$ later lecture
Summary of part 2

- perturbative calculations beyond tree level only for quantities that are IR and collinear safe and hence dominated by large virtualities
- simplest examples: total cross sections/decay rates for colourless initial states
- for differential cross sections/distributions: can have large double logarithms from soft and collinear emissions
- for jets in final state suitable (and unsuitable) observables exist