Introduction to perturbative QCD and factorization

Part 3: factorisation

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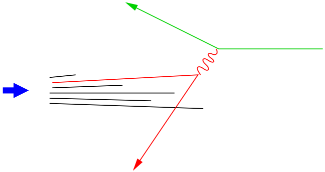
HELMHOLTZ
GEMEINSCHAFT

DESY

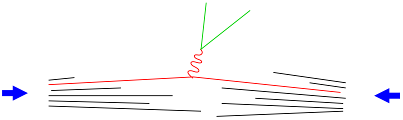
The parton model

description for deep inelastic scattering, Drell-Yan process, etc.

- fast-moving hadron
  $\approx$ set of free partons $(q, \bar{q}, g)$ with low transverse momenta
- physical cross section
  $=$ cross section for partonic process $(\gamma^* q \to q,\ q\bar{q} \to \gamma^*)$
  $\times$ parton densities



Deep inelastic scattering (DIS): $\ell p \to \ell X$



Drell-Yan: $pp \to \ell^+ \ell^- X$

Nobel prize 1980 for
Friedman, Kendall, Taylor

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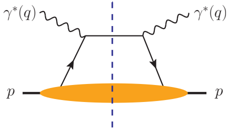
Factorisation

- implement and correct parton-model ideas in QCD
  - conditions and limitations of validity
    kinematics, processes, observables
  - corrections: partons interact
    α_s small at large scales $\rightsquigarrow$ perturbation theory
  - definition of parton densities in QCD
    derive their general properties
    make contact with non-perturbative methods

Example: inclusive DIS (deep inelastic scattering)

- $\sigma_{\text{tot}}(\gamma^* p \to X)$
  $\xrightarrow{\text{opt. theorem}} \operatorname{Im} \mathcal{A}(\gamma^* p \to \gamma^* p)$
  forward amplitude

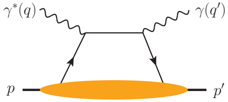


- measure in $ep \to eX$

- Bjorken limit: $Q^2 = -q^2 \to \infty$ at fixed $x_B = Q^2/(2p \cdot q)$
- $\operatorname{Im} \mathcal{A}(\gamma^* p \to \gamma^* p) =$
  hard-scattering coefficient $\otimes$ parton distribution
  - hard-scattering coefficient $\sim \operatorname{Im} \mathcal{A}(\gamma^* q \to \gamma^* q)$
    small print $\to$ later
  - parton densities (PDFs): process independent
    also appear in $pp \to \ell^+ \ell^- X$, $\gamma^* p \to \text{jet} + X$, ...

Example: DVCS (deeply virtual Compton scattering)

- exclusive cross section
  $\propto \left|\mathcal{A}(\gamma^* p \to \gamma p)\right|^2$
  square of amplitude



- measure in $ep \to ep\gamma$
- Bjorken limit: $Q^2 = -q^2 \to \infty$ at fixed x_B and $t = (p - p')^2$
- $\mathcal{A}(\gamma^* p \to \gamma p) =$
  hard-scattering coefficient $\otimes$ generalized parton distribution
  - GPD depends on momentum fractions x, ξ and on t
  - hard-scattering coefficient $\sim \mathcal{A}(\gamma^* q \to \gamma q)$

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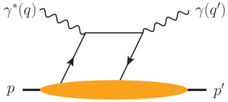

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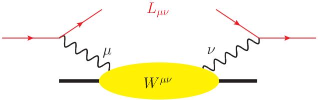
  - GPD depends on momentum fractions x, ξ and on t
  - hard-scattering coefficient $\sim$ $\mathcal{A}(\gamma^* q \to \gamma q)$ or $\mathcal{A}(\gamma^* q\bar{q} \to \gamma)$
    both cases included in $\int dx$

Interlude: DIS structure functions



- aim: separate QED/electroweak from QCD part
  - leptonic tensor $L_{\mu\nu} \propto \mathcal{A}_{\ell\to\ell+\gamma^*(\mu)} \left[\mathcal{A}_{\ell\to\ell+\gamma^*(\nu)}\right]^*$
  - hadronic tensor $W^{\mu\nu} \propto \operatorname{Im} \int d^4x\, e^{iqx} \langle p| J^\mu(x) J^\nu(0) |p\rangle$
  - $\sigma_{\ell+p\to\ell+X} \propto L_{\mu\nu} W^{\mu\nu}$
- using symmetries (parity, time reversal, current conservation) get

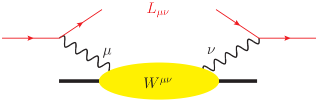
$$W^{\mu\nu}(p,q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x_B, Q^2) + \left(p^\mu - \frac{pq}{q^2}\, q^\mu\right)\left(p^\nu - \frac{pq}{q^2}\, q^\nu\right) F_2(x_B, Q^2)$$

for unpolarised proton and electromagnetic current

$\rightsquigarrow$ $\sigma_{\ell+p\to\ell+X}$ expressed through F_1 and F_2

- analogs for SIDIS $\ell + p \to \ell + h + X$, Drell-Yan, etc.

Interlude: DIS structure functions



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for unpolarised proton and electromagnetic current

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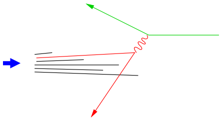
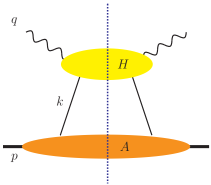
- valid in any kinematics
  no reference to factorisation
  do not confuse structure functions with parton distributions

Factorisation: physics idea and technical implementation

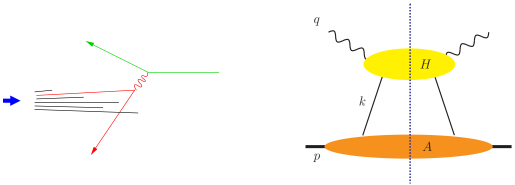




- idea: separation of physics at different scales
  - high scales: quark-gluon interactions
    $\rightsquigarrow$ compute in perturbation theory
  - low scale: proton $\to$ quarks, antiquarks, gluons
    $\rightsquigarrow$ parton densities

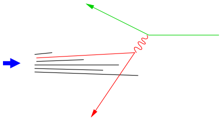
requires hard momentum scale in process
large photon virtuality $Q^2 = -q^2$ in DIS

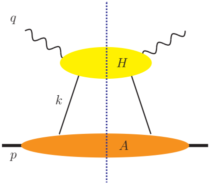
Factorisation: physics idea and technical implementation



- implementation: separate process into
  - "hard" subgraph H with particles far off-shell
    compute in perturbation theory
  - "collinear" subgraph A with particles moving along proton
    turn into definition of parton density

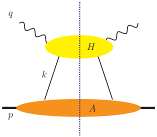
Factorisation: physics idea and technical implementation





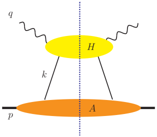
- note difference with high-energy/small x factorization
  - separate dynamics according to rapidity (not virtuality) of particles
  - overlap of two factorization schemes if have strong ordering in rapidity and virtuality

Collinear expansion



- graph gives $\int d^4k\, H(k)A(k)$; simplify further
- light-cone coordinates $\rightsquigarrow$ blackboard

Collinear expansion



- graph gives $\int d^4k\, H(k)A(k)$; simplify further
- in hard graph neglect small components of external lines
  $\rightsquigarrow$ Taylor expansion

$$H(k^+, k^-, k_T) = H(k^+, 0, 0) + \text{corrections}$$

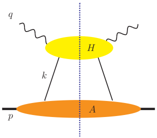
  $\rightsquigarrow$ loop integration greatly simplifies:

$$\int d^4k\; H(k)\, A(k) \approx \int dk^+\, H(k^+, 0, 0)\; \int dk^- d^2k_T\; A(k^+, k^-, k_T)$$

- in hard scattering treat incoming/outgoing partons as exactly collinear $(k_T = 0)$ and on-shell $(k^- = 0)$
- in collin. matrix element integrate over k_T and virtuality
  $\rightsquigarrow$ collinear (or k_T integrated) parton densities
  only depend on $k^+ = xp^+$

further subtleties related with spin of partons, not discussed here

Definition of parton distributions



- matrix elements of quark/gluon operators

$$f_q(x) = \int \frac{dz^-}{2\pi}\, e^{ixp^+z^-} \left\langle p \middle| \bar{q}(0)\, \tfrac{1}{2}\gamma^+ W[0,z]\, q(z) \middle| p \right\rangle \Big|_{z^+=0,\; z_T=0}$$

  $q(z) =$ quark field operator: annihilates quark

  $\bar{q}(0) =$ conjugate field operator: creates quark

  $\frac{1}{2}\gamma^+$ sums over quark spin

  $\int \frac{dz^-}{2\pi}\, e^{ixp^+z^-}$ projects on quarks with $k^+ = xp^+$

  $W[0,z] =$ Wilson line, makes product of fields gauge invariant $\rightsquigarrow$ later

- analogous definitions for polarised quarks, antiquarks, gluons
- analysis of factorisation used Feynman graphs
  but here provide non-perturbative definition

Lowest order results for DIS and DVCS

- hard-scattering part of handbag graphs:

kinematics → blackboard

Lowest order results for DIS and DVCS

- hard-scattering part of handbag graphs:

$$\frac{1}{x - x_B + i\varepsilon} + \{\text{crossed graph}\} = \mathrm{PV}\,\frac{1}{x - x_B} - i\pi\delta(x - x_B) + \{\text{crossed graph}\}$$

- for DIS:

$$\sigma_{\text{tot}} \propto \operatorname{Im}\mathcal{A}(\gamma_T^* p \to \gamma_T^* p) = \sum_q (ee_q)^2 \big[q(x_B) + \bar{q}(x_B)\big]$$

$$\mathcal{A}(\gamma_L^* p \to \gamma_L^* p) = 0$$

$$2x_B F_1 = F_2 = x_B \sum_q e_q^2 \,\big[q(x_B) + \bar{q}(x_B)\big]$$

Lowest order results for DIS and DVCS

- hard-scattering part of handbag graphs:

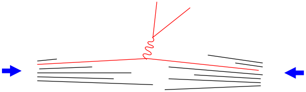
$$\frac{1}{x - x_B + i\varepsilon} + \{\text{crossed graph}\} = \mathrm{PV}\,\frac{1}{x - x_B} - i\pi\delta(x - x_B) + \{\text{crossed graph}\}$$

- for DVCS:

$$\mathcal{A}(\gamma_T^* p \to \gamma_T p) = \sum_q (e e_q)^2 \left[\mathrm{PV} \int dx \, \frac{\mathsf{GPD}(x, x_B, t)}{x_B - x} + i\pi\, \mathsf{GPD}(x_B, x_B, t) \right] + \{\text{c.g.}\}$$

Factorisation for pp collisions

- example: Drell-Yan process $pp \to \gamma^* + X \to \mu^+\mu^- + X$ where $X =$ any number of hadrons
- one parton distribution for each proton $\times$ hard scattering $\rightsquigarrow$ deceptively simple physical picture



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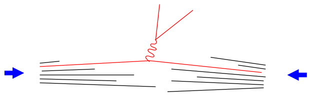
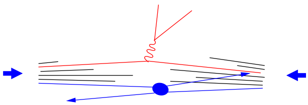




- "spectator" interactions produce additional particles
  which are also part of unobserved system X ("underlying event")
- need not calculate this thanks to unitarity
  as long as cross section/observable sufficiently inclusive
- but must calculate/model if want more detail on the final state

More complicated final states

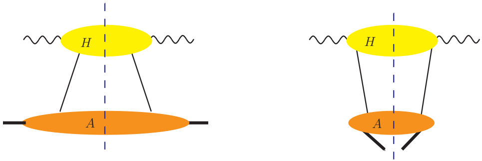
- production of W, Z or other colourless particle (Higgs, etc) same treatment as Drell-Yan
- jet production in ep or pp: hard scale provided by p_T
- heavy quark production: hard scale is m_c, m_b, m_t

Importance of factorisation concept

- describe high-energy processes: study electroweak physics, search for new particles, e.g.
    - discovery of top quark at Tevatron ($p + \bar{p}$ at $\sqrt{s} = 1.8\,\text{TeV}$)
    - measurement of W mass at Tevatron and LHC
    - determination of Higgs boson properties at LHC
- determine parton densities as a tool to make predictions and to learn about proton structure
    - require many processes to disentangle quark flavors and gluons

Fragmentation

- cross DIS $eh \to e + X$ to $e^+e^- \to \bar{h} + X$
  i.e., $\gamma^* h \to X$ to $\gamma^* \to \bar{h} + X$



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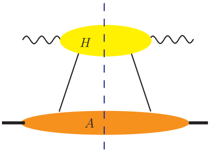
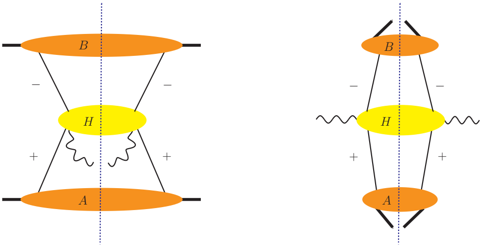


- or Drell-Yan $h_1 h_2 \to \gamma^* + X$ to $\gamma^* \to \bar{h}_1 \bar{h}_2 + X$



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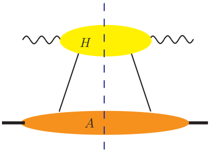
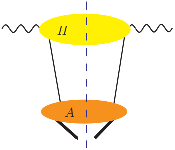
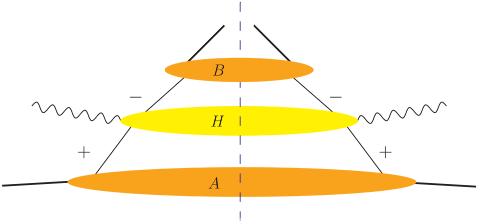




- or SIDIS $eh_1 \to eh_2 + X$



Fragmentation functions

- replace parton density $k^+ = xp^+$

$$f(x) = \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+ x} \langle h|\bar{q}(0)\gamma^+ W(0,\xi^-)\, q(\xi^-)|h\rangle$$

$$= \sum_X \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+ x}$$

$$\times \sum_X \big\langle h \big| (\bar{q}(0)\gamma^+)_\alpha W(0,\infty) \big| X \big\rangle \big\langle X \big| W(\infty,\xi^-)\, q_\alpha(\xi^-) \big| h \big\rangle$$

by fragmentation function $p^+ = zk^+$

$$D(z) = \frac{1}{2N_c\, z} \int \frac{d\xi^-}{4\pi}\, e^{i\xi^- p^+/z}$$

$$\times \sum_X \big\langle 0 \big| W(\infty,\xi^-)\, q_\alpha(\xi^-) \big| \bar{h} X \big\rangle \big\langle \bar{h} X \big| (\bar{q}(0)\gamma^+)_\alpha W(0,\infty) \big| 0 \big\rangle$$

A closer look at one-loop corrections

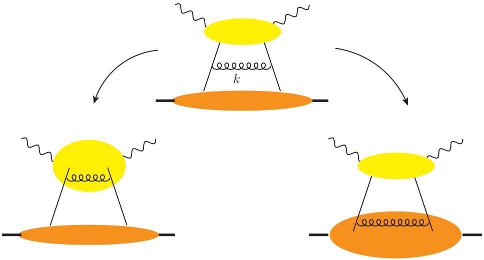
- example: DIS

- UV divergences removed by standard renormalisation
- soft divergences cancel in sum over graphs
- collinear div. do not cancel, have integrals

$$\int_0 \frac{dk_T^2}{k_T^2}$$

what went wrong?

- hard graph should not contain internal collinear lines
  collinear graph should not contain hard lines
- must not double count $\rightsquigarrow$ factorisation scale μ



- with cutoff: take $k_T > \mu$ take $k_T < \mu$
  $1/\mu \sim$ transverse resolution

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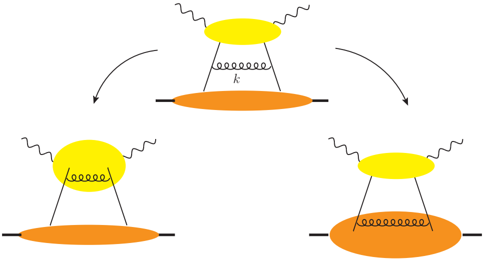
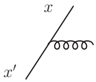


- with cutoff: take $k_T > \mu$ — take $k_T < \mu$
  $1/\mu \sim$ transverse resolution
- in dim. reg.:
  subtract collinear divergence — subtract ultraviolet div.

The evolution equations

- DGLAP equations

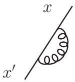
$$\frac{d}{d\log\mu^2}\,f(x,\mu) = \int_x^1 \frac{dx'}{x'}\,P\Big(\frac{x}{x'}\Big)\,f(x',\mu) \;=\; \big(P\otimes f(\mu)\big)(x)$$



- $P =$ splitting functions
  - have perturbative expansion

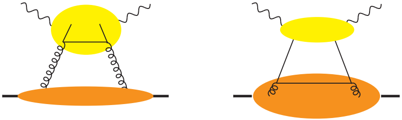
$$P(x) = \alpha_s(\mu)\,P^{(0)}(x) + \alpha_s^2(\mu)\,P^{(1)}(x) + \alpha_s^3(\mu)\,P^{(2)}(x)\ldots$$

known to 3 loops Moch, Vermaseren, Vogt 2004



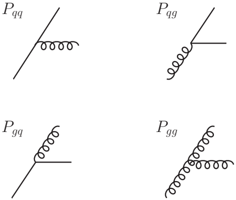
  - contains terms $\propto \delta(1-x)$ from virtual corrections

- quark and gluon densities mix under evolution:



- matrix evolution equation

$$\frac{d}{d\log\mu^2} f_i(x,\mu) = \sum_{j=q,\bar{q},g} \big(P_{ij} \otimes f_j(\mu)\big)(x) \qquad (i,j = q,\bar{q},g)$$



- parton content of proton depends on resolution scale μ

Factorisation formula

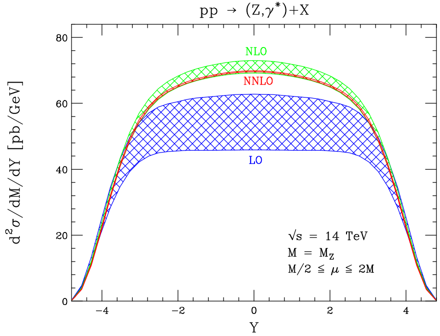
- example: $p + p \to H + X$

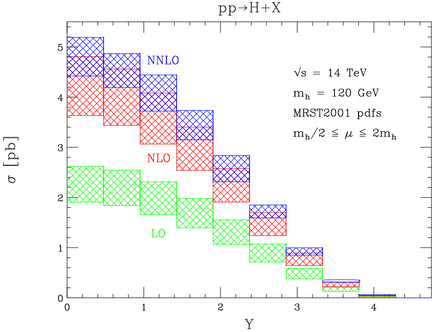
$$\sigma(p+p \to H+X) = \sum_{i,j=q,\bar{q},g} \int dx_i\, dx_j\; f_i(x_i,\mu_F)\, f_j(x_j,\mu_F) \times \hat{\sigma}_{ij}\big(x_i, x_j, \alpha_s(\mu_R), \mu_R, \mu_F, m_H\big) + \mathcal{O}\left(\frac{\Lambda^2}{m_H^4}\right)$$

  - $\hat{\sigma}_{ij} =$ cross section for hard scattering $i + j \to H + X$
    m_H provides hard scale
  - $\mu_R =$ renormalisation scale, $\mu_F =$ factorisation scale
    may take different or equal
  - μ_F dependence in C and in f cancels up to higher orders in α_s
    similar discussion as for μ_R dependence
  - accuracy: α_s expansion and power corrections $\mathcal{O}(\Lambda^2/m_H^2)$
- can make σ and $\hat{\sigma}$ differential in kinematic variables, e.g. p_T of H

Scale dependence

examples: rapidity distributions in Z/γ^* and in Higgs production





Anastasiou, Dixon, Melnikov, Petriello, hep-ph/0312266

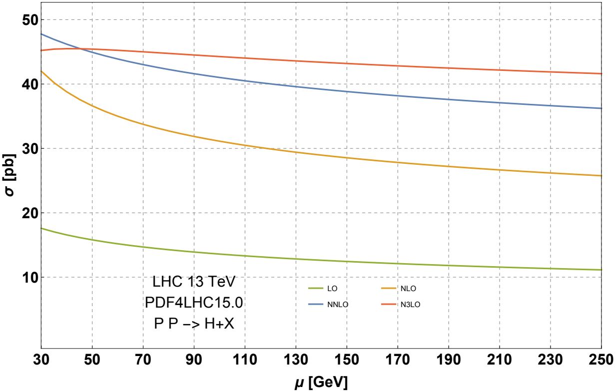
Anastasiou, Melnikov, Petriello, hep-ph/0501130

$\mu_F = \mu_R = \mu$ varied within factor 1/2 to 2

Scale dependence

example: inclusive Higgs production

$pp \to H + X$



Mistlberger, arXiv:1802.00833

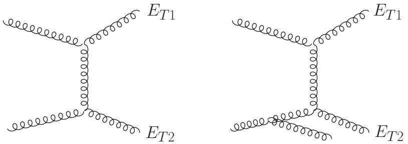
LO, NLO, and higher

- instead of varying scale(s) may estimate higher orders by comparing N^{n}LO result with N^{n-1}LO
- caveat: comparison NLO vs. LO may not be representative for situation at higher orders

  often have especially large step from LO to NLO

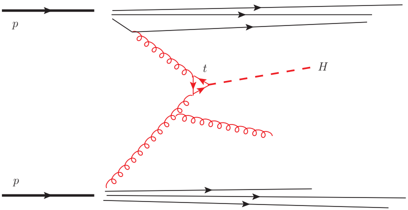
  - certain types of contribution may first appear at NLO
    e.g. terms with gluon density $g(x)$ in DIS, $pp \to Z + X$, etc.
  - final state at LO may be too restrictive

    e.g. in $\dfrac{d\sigma}{dE_{T1}\, dE_{T2}}$ for dijet production



General purpose event generators e.g. Herwig, Pythia, Sherpa

- build on structure of factorisation formulae e.g. for $pp \to H + g + X$
- but compute fully specified events, i.e. no "$+X$"
  schematically:



- ingredients:
  - parton densities and hard-scattering matrix elements

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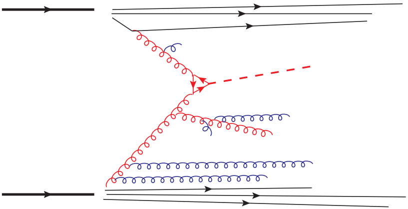

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  - parton showers: small-angle radiation from partons in initial and final state (in perturbative region)

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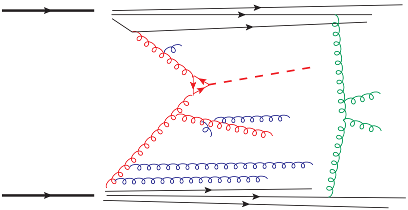

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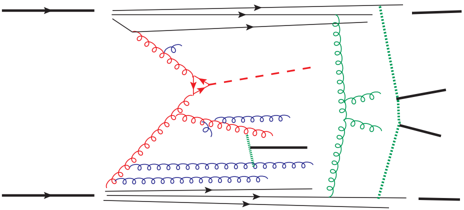

- ingredients:
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  - parton showers: small-angle radiation from partons in initial and final state (in perturbative region)
  - models for multiparton interactions and hadronisation

Summary of part 3

Factorisation

- implements ideas of parton model in QCD
  - perturbative corrections (NLO, NNLO, . . .)
  - field theoretical def. of parton densities
    $\rightsquigarrow$ bridge to non-perturbative QCD
- valid for sufficiently inclusive observables
  and up to power corrections in Λ/Q or $(\Lambda/Q)^2$
  which are in general not calculable
- must in a consistent way
  - remove collinear kinematic region in hard scattering
  - remove hard kinematic region in parton densities
    $\leftrightarrow$ UV renormalisation

  procedure introduces factorisation scale μ_F
  - separates "collinear" from "hard", "object" from "probe"

Summary of part 3

Factorisation

- implements ideas of parton model in QCD
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    $\rightsquigarrow$ bridge to non-perturbative QCD
- valid for sufficiently inclusive observables
  and up to power corrections in Λ/Q or $(\Lambda/Q)^2$
  which are in general not calculable
- theoretical backbone for simulating hard processes
  in many event generators

Factorisation at work