QCD and hadron structure
Part 6: Double parton scattering

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Hadron-hadron collisions

- standard description based on factorization formulae

\[
\text{cross sect} = \text{parton distributions} \times \text{parton-level cross sect}
\]

- factorization formulae are for inclusive cross sections \( pp \rightarrow Y + X \)
  where \( Y = \) produced in parton-level scattering, specified in detail
  \( X = \) summed over, no details
Hadron-hadron collisions

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- factorization formulae are for inclusive cross sections \( pp \rightarrow Y + X \)
  
  where \( Y \) = produced in parton-level scattering, specified in detail
  
  \( X \) = summed over, no details

- also have interactions between “spectator” partons
  
  their effects cancel in inclusive cross sections thanks to unitarity
  
  but they affect the final state \( X \)

- spectator interactions can be soft (low \( p_T \) scattering) \( \rightsquigarrow \) underlying event
  
  or hard \( \rightsquigarrow \) multiparton interactions

- here: concentrate on hard double parton scattering (DPS)
Single vs. double hard scattering

▶ example: prod’n of two gauge bosons, transverse momenta $q_1$ and $q_2$

single scattering:

$|q_1|$ and $|q_1| \sim$ hard scale $Q^2$

$|q_1 + q_2| \ll Q^2$

▶ for transv. mom. $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{single}}}{d^2q_1 d^2q_2} \sim \frac{d\sigma_{\text{double}}}{d^2q_1 d^2q_2} \sim \frac{1}{Q^4 \Lambda^2}$$

but single scattering populates larger phase space:

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \gg \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}$$
Single vs. double hard scattering

▶ example: prod’n of two gauge bosons, transverse momenta \( q_1 \) and \( q_2 \)

\[
\begin{align*}
|q_1| & \text{ and } |q_1| \sim \text{ hard scale } Q^2 \\
|q_1 + q_2| & \ll Q^2 \\
\end{align*}
\]

▶ double scattering favored at small \( x \) (high energies):

densities for two partons rise faster than for single parton
A numerical example

gauge boson pair production

\[ \begin{align*}
W^+ & \rightarrow W^+ W^+ \\
W^+ & \rightarrow W^+ W^+ 
\end{align*} \]

single scattering:
\[ qq \rightarrow qq + W^+ W^+ \]
suppressed by \( \alpha_s^2 \)

integrated cross section

\[ \sigma \text{(nb)} \]

CM energy (TeV)

J Gaunt et al, arXiv:1003.3953
Feynman graphs: momentum vs. distance

- large (plus or minus) momenta of partons $x_i p, \bar{x}_i \bar{p}$ fixed by final state exactly as for single hard scattering
- transverse parton momenta not the same in amplitude $A$ and in $A^*$
  cross section $\propto \int d^2 r \ F(x_i, k_i, r) F(\bar{x}_i, \bar{k}_i, -r)$
- Fourier trf. to impact parameter: $F(x_i, k_i, r) \rightarrow F(x_i, k_i, y)$
  cross section $\propto \int d^2 y \ F(x_i, k_i, y) F(\bar{x}_i, \bar{k}_i, y)$
- interpretation: $y = \text{transv. dist. between two scattering partons}$
  $= \text{equal in both colliding protons}$

\[ q^2 \]
\[ q^1 \]
\[ k^1 - 1 \frac{r}{2} \]
\[ k^2 + 1 \frac{r}{2} \]
\[ k^2 - 1 \frac{r}{2} \]
\[ k^1 + 1 \frac{r}{2} \]
\[ r \]
\[ \bar{r} \]
\[ p \]
\[ \bar{p} \]
\[ \Rightarrow r + \bar{r} = 0 \]

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DPS cross section: collinear factorisation

\[
\frac{d\sigma_{\text{double}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2y \ F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)
\]

- \(C = \) combinatorial factor
- \(\hat{\sigma}_i = \) parton-level cross sections
- \(F(x_1, x_2, y) = \) double parton distribution (DPD)
- \(y = \) transv. distance between partons

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make \(\hat{\sigma}_i\) differential in further variables (e.g. for jet pairs)
- can extend \(\hat{\sigma}_i\) to higher orders in \(\alpha_s\)
  get usual convolution integrals over \(x_i\) in \(\hat{\sigma}_i\) and \(F\)

Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2012
DPS cross section: TMD factorisation

- for measured transv. momenta

\[
\frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, d^2q_1 \, dx_2 \, d\bar{x}_2 \, d^2q_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \\
\times \int \frac{d^2z_1}{(2\pi)^2} \frac{d^2z_2}{(2\pi)^2} e^{-i(z_1 q_1 + z_2 q_2)} \int d^2y \, F(x_i, z_i, y) F(\bar{x}_i, z_i, y)
\]

- \(F(x_i, z_i, y) = \) double-parton TMDs
  \(z_i = \) Fourier conjugate to parton transverse mom. \(k_i\)

- operator definition as for TMDs: schematically have

\[
F(x_i, z_i, y) = \mathcal{F}T_{z_i \to x_i p^+} \langle p| \bar{q}(-\frac{1}{2} z_2) \Gamma_2 q(\frac{1}{2} z_2) \bar{q}(y - \frac{1}{2} z_1) \Gamma_1 q(y + \frac{1}{2} z_1) |p\rangle
\]

- to be completed by renormalisation, Wilson lines, soft factors
- essential for studying factorisation, scale and rapidity dependence

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QCD and hadron structure
DPS cross section: TMD factorisation

- for measured transv. momenta

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\frac{d\sigma_{\text{DPS}}}{dx_1 \, d\bar{x}_1 \, d^2q_1 \, dx_2 \, d\bar{x}_2 \, d^2q_2} = \frac{1}{C} \, \hat{\sigma}_1 \, \hat{\sigma}_2 \\
\quad \times \int \frac{d^2z_1}{(2\pi)^2} \, \frac{d^2z_2}{(2\pi)^2} \, e^{-i(z_1 q_1 + z_2 q_2)} \int d^2y \, F(x_i, z_i, y) \, F(\bar{x}_i, z_i, y)
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  \( z_i = \) Fourier conjugate to parton transverse mom. \( k_i \)

- operator definition as for TMDs: schematically have

\[
F(x_i, z_i, y) = \mathcal{FT} \, \left\langle p \right| \bar{q}(-\frac{1}{2}z_2) \Gamma_2 q(\frac{1}{2}z_2) \, \bar{q}(y - \frac{1}{2}z_1) \Gamma_1 q(y + \frac{1}{2}z_1) |p\rangle
\]

- to be completed by renormalisation, Wilson lines, soft factors
- essential for studying factorisation, scale and rapidity dependence

\[ \text{M Buffing, MD, T Kasemets 2017} \]

- analogous def for collinear distributions \( F(x_i, y) \)
  
  \( \Rightarrow \) not a twist-four operator but product of two twist-two operators
Pocket formula

- make simplest possible assumptions
- if two-parton density factorises as
  \[ F(x_1, x_2, y) = f(x_1) f(x_2) G(y) \]
  where \( f(x_i) = \) usual PDF
- if assume same \( G(y) \) for all parton types
  then cross sect. formula turns into
  \[
  \frac{d\sigma_{\text{double}}}{dx_1 \, \bar{x}_1 \, dx_2 \, \bar{x}_2} = \frac{1}{C} \frac{d\sigma_1}{dx_1 \, \bar{x}_1} \frac{d\sigma_2}{dx_2 \, \bar{x}_2} \frac{1}{\sigma_{\text{eff}}}
  \]
  with \( 1/\sigma_{\text{eff}} = \int d^2 y \, G(y)^2 \)
  \( \approx \) scatters are completely independent
- used in bulk of phenomenological estimates
- underlies modeling of MPI in Pythia, Herwig, . . . (with some refinements)
- fails if any of the above assumptions is invalid
  or if original cross sect. formula misses important contributions
Experimental investigations (incomplete)

- CDF 4 jets (1993)
- CDF $\gamma + 3$ jets (1997)
- CDF reanalysis, Bahr et al (2013)
- D0 $\gamma + 3$ jets (2009)
- D0 $\gamma + 3$ jets (2014)
- D0 $\gamma + 2$ jets + b/c jet (2014)
- D0 2 $\gamma + 2$ jets (2015)
- CMS W + 2 jets (2013)
- ATLAS W + 2 jets (2013)
- ATLAS 4 jets (2016)
- LHCb $\Upsilon + D$ (2015)
- LHCb $J/\Psi + J/\Psi$ (2011)
- D0 $J/\Psi + J/\Psi$ (2014)
- D0 $J/\Psi + \Upsilon$ (2015)
- ATLAS $J/\Psi + J/\Psi$ (2016)

Other channels:
- double open charm $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$  
  LHCb 2012
- $W + J/\Psi$, $Z + J/\Psi$  
  ATLAS 2014, 2015
- $\Upsilon + \Upsilon$ (estimate $\sigma_{\text{eff}} \approx 2.2 \div 6.6 \text{ mb}$)  
  CMS 2016
- same-sign $WW$ (LHC run 2)  
  CMS 2017
Parton correlations

- If neglect correlations between two partons
  \[ F(x_1, x_2, y) = \int d^2 b \ f(x_2, b) f(x_1, b + y) \]

  where \( f(x_i, b) = \) impact parameter dependent single-parton density

- And if neglect correlations between \( x \) and \( y \) of single parton
  \[ f(x_i, b) = f(x_i) F(b) \]

  then
  \[ F(x_1, x_2, y) = f(x_1) f(x_2) \int d^2 b \ F(b) F(b + y) \]
Parton correlations

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  \[ F(x_1, x_2, y) = \int d^2 b \ f(x_2, b) \ f(x_1, b + y) \]
  where \( f(x_i, b) = \text{impact parameter dependent single-parton density} \)

- and if neglect correlations between \( x \) and \( y \) of single parton
  \[ f(x_i, b) = f(x_i) F(b) \]

- then \[ F(x_1, x_2, y) = f(x_1) f(x_2) \int d^2 b \ F(b) \ F(b + y) \]

- information on \( f(x, b) \) from study of GPDs and elastic form factors

  ⊕ measurements of double parton scattering
  → complete independence between two partons disfavored

Parton correlations

at certain level of accuracy expect correlations between

▶ $x_1$ and $x_2$ of partons
  - most obvious: energy conservation $\Rightarrow x_1 + x_2 \leq 1$
  - significant $x_1 - x_2$ correlations found in quark models
    Chang, Manohar, Waalewijn 2012; Rinaldi et al 2013–16
    Broniowski et al 2013–16; Kasemets, Mukherjee 2016

• $x_i$ and $y$
  even for single partons see correlations between $x$ and $b$ distribution
  - HERA results on $\gamma p \rightarrow J/\Psi p$ give
    $\langle b^2 \rangle \propto \text{const} + 4\alpha' \log(1/x)$ with $4\alpha' \approx (0.16 \text{ fm})^2$
    for gluons with $x \sim 10^{-3}$
  - lattice calculations of $x^0$, $x^1$, $x^2$ moments
    $\rightarrow$ strong decrease of $\langle b^2 \rangle$ with $x$ above $\sim 0.1$

plausible to expect similar correlations in double parton distributions
even if two partons not uncorrelated
Consequence for multiple interactions

- indications for decrease of \( \langle y^2 \rangle \) with \( x \)
- if interaction 1 produces high-mass system
  - have large \( x_1, \bar{x}_1 \)
  - smaller \( y \) → more central collision
  - secondary interactions enhanced

Frankfurt, Strikman, Weiss 2003

studies in Pythia: Corke, Sjöstrand 2011; Blok, Gunnellini 2015
Colour structure

- quark lines in amplitude and its conjugate can couple to color singlet or octet:

\[
1F \rightarrow (\bar{q}_2 \Gamma q_2) (\bar{q}_1 \Gamma q_1) \quad 8F \rightarrow (\bar{q}_2 t^\alpha q_2) (\bar{q}_1 t^\alpha q_1)
\]

- \(8F\) describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)
- for two-gluon dist’s more color structures
- color correlations suppressed by Sudakov logarithms

... but not necessarily negligible for moderately hard scales

Manohar, Waalewijn arXiv:1202:3794

\[U = \text{Sudakov factor, } Q = \text{hard scale}\]
Double parton scattering: ultraviolet problem

\[
\frac{d\sigma_{\text{DPS}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 y \ F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)
\]

for \( y \ll 1/\Lambda \) can compute

\[
F(x_1, x_2, y) \sim \frac{1}{y^2} \text{ splitting fct \( \otimes \) usual PDF}
\]

gives strong correlations in colour and spin
Double parton scattering: ultraviolet problem

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\frac{d\sigma_{\text{DPS}}}{dx_1 d\bar{x}_1 dx_2 d\bar{x}_2} = \frac{1}{C} \hat{\sigma}_1 \hat{\sigma}_2 \int d^2 y \ F(x_1, x_2, y) F(\bar{x}_1, \bar{x}_2, y)
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▶ for \( y \ll 1/\Lambda \) can compute

\[
F(x_1, x_2, y) \sim \frac{1}{y^2} \text{ splitting fct } \otimes \text{ usual PDF}
\]

gives UV divergent cross section \( \propto \int d^2 y / y^4 \)
in fact, formula not valid for \( |y| \sim 1/Q \)

▶ problem also for two-parton TMDs

UV divergences logarithmic instead of quadratic
... and more problems

- **double counting** problem between double scattering with splitting (1v1) and single scattering at loop level
  
  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- also have graphs with splitting in one proton only: “2v1”

\[ \sim \int \frac{d^2 y}{y^2} \times F_{\text{int}}(x_1, x_2, y) \]

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015
A consistent solution

MD, J. Gaunt, K. Schönwald 2017

regulate DPS: \( \sigma_{\text{DPS}} \propto \int d^2 y \ \Phi^2(\nu y) \ F(x_1, x_2, y) \ F(\bar{x}_1, \bar{x}_2, y) \)

- \( \Phi \to 0 \) for \( u \to 0 \) and \( \Phi \to 1 \) for \( u \to \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
- cutoff scale \( \nu \sim Q \)
- \( F(x_1, x_2, y) \) has both splitting and 'intrinsic' contributions

analogous regulator for transverse-momentum dependent DPDs

keep definition of DPDs as operator matrix elements

cutoff in \( y \) does not break symmetries that haven't already been broken
A consistent solution

regulate DPS: \( \sigma_{\text{DPS}} \propto \int d^2y \ \Phi^2(\nu y) \ F(x_1, x_2, y) \ F(\bar{x}_1, \bar{x}_2, y) \)

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analogous regulator for transverse-momentum dependent DPDs

full cross section: \( \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}} \)

- subtraction \( \sigma_{\text{sub}} \) to avoid double counting:
  \( = \sigma_{\text{DPS}} \) with \( F \) computed for small \( y \) in fixed order perturb. theory
  much simpler computation than \( \sigma_{\text{SPS}} \) at given order
A consistent solution

MD, J. Gaunt, K. Schönwald 2017

- regulate DPS: \( \sigma_{\text{DPS}} \propto \int d^2 y \, \Phi^2(\nu y) \, F(x_1, x_2, y) \, F(\bar{x}_1, \bar{x}_2, y) \)
  - \( \Phi \to 0 \) for \( u \to 0 \) and \( \Phi \to 1 \) for \( u \to \infty \), e.g. \( \Phi(u) = \theta(u - 1) \)
  - cutoff scale \( \nu \sim Q \)
  - \( F(x_1, x_2, y) \) has both splitting and 'intrinsic' contributions

- analogous regulator for transverse-momentum dependent DPDs

- full cross section: \( \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} (1v1 + 2v1) + \sigma_{\text{SPS}} + \sigma_{\text{tw2} \times \text{tw4}} \)
  - subtraction \( \sigma_{\text{sub}} \) to avoid double counting:
    - \( = \sigma_{\text{DPS}} \) with \( F \) computed for small \( y \) in fixed order perturb. theory
    - much simpler computation than \( \sigma_{\text{SPS}} \) at given order
  - can also include twist 2 \( \times \) twist 4 contribution and double counting subtraction for 2v1 term
Subtraction formalism at work

\[ \sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}} \]

- for \( y \sim 1/Q \) have \( \sigma_{\text{DPS}} \approx \sigma_{\text{sub}} \)
  because pert. computation of \( F \) gives good approx. at considered order
  \[ \Rightarrow \sigma \approx \sigma_{\text{SPS}} \]
  dependence on \( \Phi(\nu y) \) cancels between \( \sigma_{\text{DPS}} \) and \( \sigma_{\text{sub}} \)

- for \( y \gg 1/Q \) have \( \sigma_{\text{sub}} \approx \sigma_{\text{SPS}} \)
  because DPS approximations work well in box graph
  \[ \Rightarrow \sigma \approx \sigma_{\text{DPS}} \]
  with regulator fct. \( \Phi(\nu y) \approx 1 \)

- same argument for 2v1 term and \( \sigma_{\text{tw2} \times \text{tw4}} \) (were neglected above)

- subtraction formalism works order by order in perturb. theory

  J. Collins, Foundations of Perturbative QCD, Chapt. 10
Double counting in TMD factorisation for DPS

- left and right box can independently be collinear or hard:
  → DPS, DPS/SPS interference and SPS
- get nested double counting subtractions

M Buffing, MD, T Kasemets 2017
Summary

- multiparton interactions ubiquitous in hadron-hadron collisions
  - multiple hard scattering often suppressed, but not necessarily
    - in specific kinematics
    - for multi-differential cross sections, high-multiplicity final states
- double hard scattering depends on detailed hadron structure
  - transverse spatial distribution
  - different correlation and interference effects
- short-distance singularity and double counting with single scattering:
  have consistent solution using cutoff and subtraction terms
- subject of high interest for
  - control over final states at LHC
  - understanding QCD dynamics and hadron structure