QCD and hadron structure

Part 6: Double parton scattering

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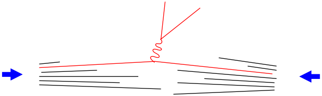
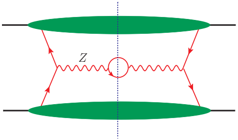
Ecole Joliot Curie 2018

Hadron-hadron collisions

- standard description based on factorization formulae

  cross sect = parton distributions × parton-level cross sect





- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where $Y =$ produced in parton-level scattering, specified in detail
  $X =$ summed over, no details

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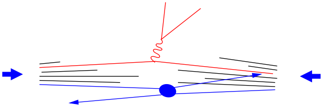
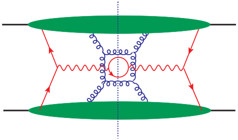
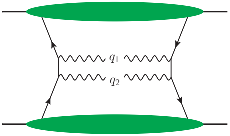


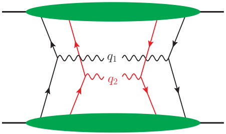


- factorization formulae are for inclusive cross sections $pp \to Y + X$
  where $Y =$ produced in parton-level scattering, specified in detail
  $X =$ summed over, no details
- also have interactions between "spectator" partons
  their effects cancel in inclusive cross sections thanks to unitarity
  but they affect the final state X
- spectator interactions can be soft (low p_T scattering) $\rightsquigarrow$ underlying event
  or hard $\rightsquigarrow$ multiparton interactions
- here: concentrate on hard double parton scattering (DPS)

Single vs. double hard scattering

- example: prod'n of two gauge bosons, transverse momenta $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$





single scattering:

$|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_1| \sim$ hard scale Q^2

$|\boldsymbol{q}_1 + \boldsymbol{q}_2| \ll Q^2$

double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_1| \ll Q^2$

- for transv. mom. $\sim \Lambda \ll Q$:

$$\frac{d\sigma_{\text{single}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{d\sigma_{\text{double}}}{d^2\boldsymbol{q}_1\, d^2\boldsymbol{q}_2} \sim \frac{1}{Q^4\,\Lambda^2}$$

but single scattering populates larger phase space :

$$\sigma_{\text{single}} \sim \frac{1}{Q^2} \gg \sigma_{\text{double}} \sim \frac{\Lambda^2}{Q^4}$$

Single vs. double hard scattering

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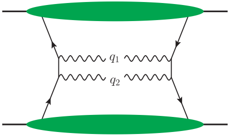

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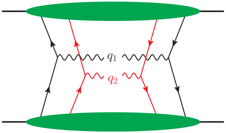


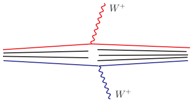
double scattering:

both $|\boldsymbol{q}_1|$ and $|\boldsymbol{q}_1| \ll Q^2$

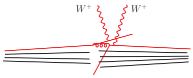
- double scattering favored at small x (high energies):
  densities for two partons rise faster than for single parton

A numerical example

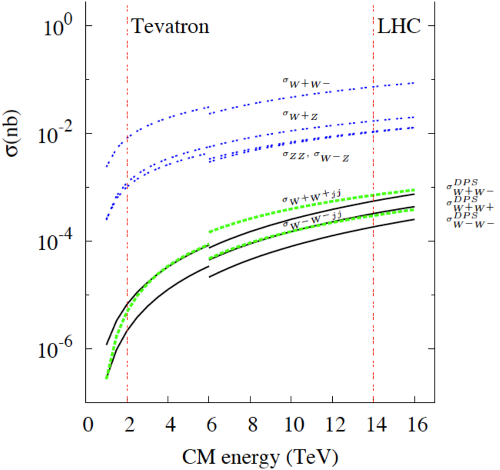
gauge boson pair production



single scattering:
$qq \to qq + W^+W^+$
suppressed by α_s^2

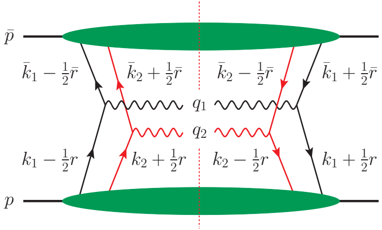


integrated cross section



J Gaunt et al, arXiv:1003.3953

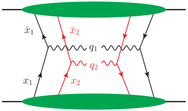
Feynman graphs: momentum vs. distance



momentum conservation $\Rightarrow r + \bar{r} = 0$

- large (plus or minus) momenta of partons $x_i p$, $\bar{x}_i \bar{p}$ fixed by final state exactly as for single hard scattering
- transverse parton momenta not the same in amplitude $\mathcal{A}$ and in $\mathcal{A}^*$
  cross section $\propto \int d^2\boldsymbol{r}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{r}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, -\boldsymbol{r})$
- Fourier trf. to impact parameter: $F(x_i, \boldsymbol{k}_i, \boldsymbol{r}) \to F(x_i, \boldsymbol{k}_i, \boldsymbol{y})$
  cross section $\propto \int d^2\boldsymbol{y}\, F(x_i, \boldsymbol{k}_i, \boldsymbol{y}) F(\bar{x}_i, \bar{\boldsymbol{k}}_i, \boldsymbol{y})$
- interpretation: $\boldsymbol{y}$ = transv. dist. between two scattering partons
  = equal in both colliding protons

DPS cross section: collinear factorisation



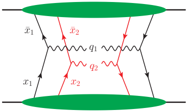
$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\; dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$

$$\begin{aligned} C &= \text{combinatorial factor} \\ \hat{\sigma}_i &= \text{parton-level cross sections} \\ F(x_1, x_2, \boldsymbol{y}) &= \text{double parton distribution (DPD)} \\ \boldsymbol{y} &= \text{transv. distance between partons} \end{aligned}$$

- follows from Feynman graphs and hard-scattering approximation
  no semi-classical approximation required
- can make $\hat{\sigma}_i$ differential in further variables (e.g. for jet pairs)
- can extend $\hat{\sigma}_i$ to higher orders in α_s
  get usual convolution integrals over x_i in $\hat{\sigma}_i$ and F

Paver, Treleani 1982, 1984; Mekhfi 1985, . . . , MD, Ostermeier, Schäfer 2012

DPS cross section: TMD factorisation



- for measured transv. momenta

$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\;\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C}\,\hat{\sigma}_1\,\hat{\sigma}_2$$
$$\times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2}\,\frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, e^{-i(\boldsymbol{z}_1\boldsymbol{q}_1+\boldsymbol{z}_2\boldsymbol{q}_2)} \int d^2\boldsymbol{y}\, F(x_i,\boldsymbol{z}_i,\boldsymbol{y})\, F(\bar{x}_i,\boldsymbol{z}_i,\boldsymbol{y})$$

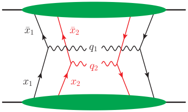
- $F(x_i,\boldsymbol{z}_i,\boldsymbol{y})$ = double-parton TMDs
  $\boldsymbol{z}_i$ = Fourier conjugate to parton transverse mom. $\boldsymbol{k}_i$
- operator definition as for TMDs: schematically have

$$F(x_i,\boldsymbol{z}_i,\boldsymbol{y}) = \underset{z_i^- \to x_i p^+}{\mathcal{FT}} \langle p|\bar{q}\big(-\tfrac{1}{2}z_2\big)\Gamma_2\, q\big(\tfrac{1}{2}z_2\big)\,\bar{q}\big(y-\tfrac{1}{2}z_1\big)\Gamma_1\, q\big(y+\tfrac{1}{2}z_1\big)|p\rangle$$

  - to be completed by renormalisation, Wilson lines, soft factors
  - essential for studying factorisation, scale and rapidity dependence

M Buffing, MD, T Kasemets 2017

DPS cross section: TMD factorisation



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$$\frac{d\sigma_{\text{DPS}}}{dx_1\, d\bar{x}_1\, d^2\boldsymbol{q}_1\; dx_2\, d\bar{x}_2\, d^2\boldsymbol{q}_2} = \frac{1}{C}\,\hat{\sigma}_1\,\hat{\sigma}_2 \times \int \frac{d^2\boldsymbol{z}_1}{(2\pi)^2}\,\frac{d^2\boldsymbol{z}_2}{(2\pi)^2}\, e^{-i(\boldsymbol{z}_1\boldsymbol{q}_1+\boldsymbol{z}_2\boldsymbol{q}_2)} \int d^2\boldsymbol{y}\, F(x_i,\boldsymbol{z}_i,\boldsymbol{y})\, F(\bar{x}_i,\boldsymbol{z}_i,\boldsymbol{y})$$

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  - to be completed by renormalisation, Wilson lines, soft factors
  - essential for studying factorisation, scale and rapidity dependence
    M Buffing, MD, T Kasemets 2017
  - analogous def for collinear distributions $F(x_i, \boldsymbol{y})$
    $\Rightarrow$ not a twist-four operator but product of two twist-two operators

Pocket formula

- make simplest possible assumptions
- if two-parton density factorises as

$$F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2)\, G(\boldsymbol{y})$$

where $f(x_i)$ = usual PDF

- if assume same $G(\boldsymbol{y})$ for all parton types
then cross sect. formula turns into

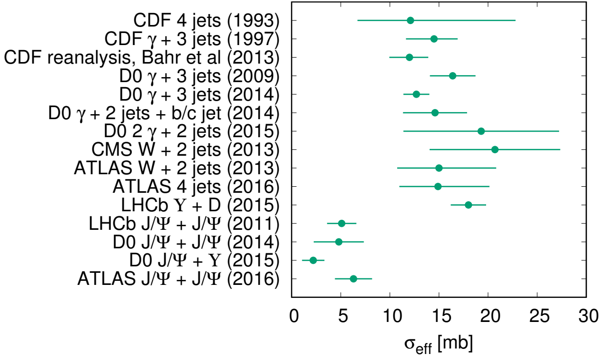
$$\frac{d\sigma_{\text{double}}}{dx_1\, d\bar{x}_1\;\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \frac{d\sigma_1}{dx_1\, d\bar{x}_1}\, \frac{d\sigma_2}{dx_2\, d\bar{x}_2}\, \frac{1}{\sigma_{\text{eff}}}$$

with $1/\sigma_{\text{eff}} = \int d^2\boldsymbol{y}\; G(\boldsymbol{y})^2$

$\rightsquigarrow$ scatters are completely independent

- used in bulk of phenomenological estimates
- underlies modeling of MPI in Pythia, Herwig, ... (with some refinements)
- fails if any of the above assumptions is invalid
or if original cross sect. formula misses important contributions

Experimental investigations (incomplete)



- other channels:
  - double open charm $C + C$ with $C = D^0, D^+, D_s^+, \Lambda_c^+$ LHCb 2012
  - $W + J/\Psi$, $Z + J/\Psi$ ATLAS 2014, 2015
  - $\Upsilon + \Upsilon$ (estimate $\sigma_{\text{eff}} \approx 2.2 \div 6.6\,\text{mb}$) CMS 2016
  - same-sign WW (LHC run 2) CMS 2017

Parton correlations

- if neglect correlations between two partons

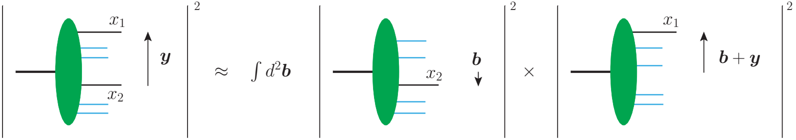
$$F(x_1, x_2, \boldsymbol{y}) = \int d^2\boldsymbol{b}\; f(x_2, \boldsymbol{b})\, f(x_1, \boldsymbol{b} + \boldsymbol{y})$$

where $f(x_i, \boldsymbol{b})$ = impact parameter dependent single-parton density

and if neglect correlations between x and $\boldsymbol{y}$ of single parton

$$f(x_i, \boldsymbol{b}) = f(x_i)\, F(\boldsymbol{b})$$

then $F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2) \int d^2\boldsymbol{b}\; F(\boldsymbol{b})\, F(\boldsymbol{b} + \boldsymbol{y})$



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then $F(x_1, x_2, \boldsymbol{y}) = f(x_1)\, f(x_2) \int d^2\boldsymbol{b}\; F(\boldsymbol{b})\, F(\boldsymbol{b} + \boldsymbol{y})$

- information on $f(x, \boldsymbol{b})$ from study of GPDs and elastic form factors
  $\oplus$ measurements of double parton scattering
  $\rightarrow$ complete independence between two partons disfavored

  cf. Calucci, Treleani 1999; Frankfurt, Strikman, Weiss 2003-04; Blok et al 2013

Parton correlations

at certain level of accuracy expect correlations between

- x_1 and x_2 of partons
  - most obvious: energy conservation $\Rightarrow$ $x_1 + x_2 \leq 1$
  - significant $x_1 - x_2$ correlations found in quark models

    Chang, Manohar, Waalewijn 2012; Rinaldi et al 2013–16
    Broniowski et al 2013–16; Kasemets, Mukherjee 2016

- x_i and $\boldsymbol{y}$

  even for single partons see correlations between x and $\boldsymbol{b}$ distribution
  - HERA results on $\gamma p \to J/\Psi\, p$ give

    $\langle \boldsymbol{b}^2 \rangle \propto$ const $+ 4\alpha' \log(1/x)$ with $4\alpha' \approx (0.16\,\text{fm})^2$

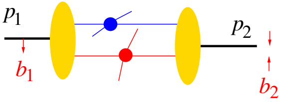
    for gluons with $x \sim 10^{-3}$
  - lattice calculations of x^0, x^1, x^2 moments

    $\to$ strong decrease of $\langle \boldsymbol{b}^2 \rangle$ with x above ~ 0.1

  plausible to expect similar correlations in double parton distributions
  even if two partons not uncorrelated

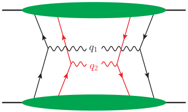
Consequence for multiple interactions

- indications for decrease of $\langle \boldsymbol{y}^2 \rangle$ with x
- if interaction 1 produces high-mass system
  $\rightarrow$ have large $x_1, \bar{x}_1$
  $\rightarrow$ smaller $\boldsymbol{y}$ $\rightarrow$ more central collision
  $\rightarrow$ secondary interactions enhanced



Frankfurt, Strikman, Weiss 2003
studies in Pythia: Corke, Sjöstrand 2011; Blok, Gunnellini 2015

Colour structure



- quark lines in amplitude and its conjugate can couple to color singlet or octet:

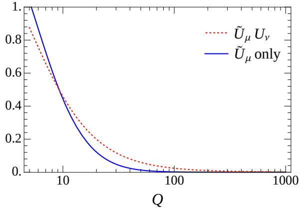
$$^{1}F \to (\bar{q}_2 \mathbb{1} q_2)\,(\bar{q}_1 \mathbb{1} q_1) \qquad\qquad ^{8}F \to (\bar{q}_2\, t^a q_2)\,(\bar{q}_1\, t^a q_1)$$

- ^{8}F describes color correlation between quarks 1 and 2 is essentially unknown (no probability interpretation as a guide)
- for two-gluon dist's more color structures
- color correlations suppressed by Sudakov logarithms — Mekhfi 1988

... but not necessarily negligible for moderately hard scales

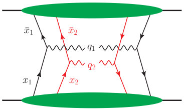
Manohar, Waalewijn arXiv:1202:3794

U = Sudakov factor, Q = hard scale



Double parton scattering: ultraviolet problem

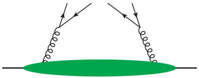
$$\frac{d\sigma_{\mathsf{DPS}}}{dx_1\, d\bar{x}_1\, dx_2\, d\bar{x}_2} = \frac{1}{C}\, \hat{\sigma}_1\, \hat{\sigma}_2 \int d^2\boldsymbol{y}\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$$



- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

  $$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

  gives strong correlations in colour and spin



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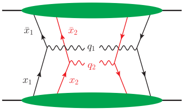
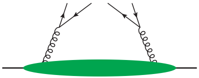


- for $\boldsymbol{y} \ll 1/\Lambda$ can compute

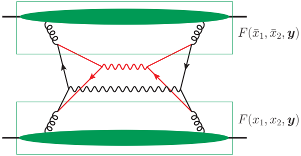
  $$F(x_1, x_2, \boldsymbol{y}) \sim \frac{1}{\boldsymbol{y}^2} \text{ splitting fct} \otimes \text{usual PDF}$$

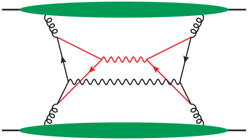
  gives UV divergent cross section $\propto \int d^2\boldsymbol{y}/\boldsymbol{y}^4$
  in fact, formula not valid for $|\boldsymbol{y}| \sim 1/Q$



- problem also for two-parton TMDs
  UV divergences logarithmic instead of quadratic

. . . and more problems





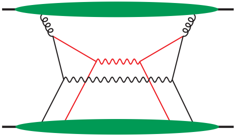
- double counting problem between double scattering with splitting (1v1) and single scattering at loop level

  MD, Ostermeier, Schäfer 2011; Gaunt, Stirling 2011; Gaunt 2012
  Blok, Dokshitzer, Frankfurt, Strikman 2011; Ryskin, Snigirev 2011, 2012
  already noted by Cacciari, Salam, Sapeta 2009

- also have graphs with splitting in one proton only: "2v1"

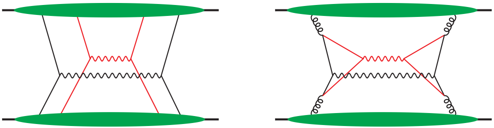
$$\sim \int d^2\boldsymbol{y}/\boldsymbol{y}^2 \times F_{\text{int}}(x_1, x_2, \boldsymbol{y})$$

B Blok et al 2011-13
J Gaunt 2012
B Blok, P Gunnellini 2015



A consistent solution

MD, J. Gaunt, K. Schönwald 2017



- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\; \Phi^2(\nu y)\; F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u - 1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions

  analogous regulator for transverse-momentum dependent DPDs
- keep definition of DPDs as operator matrix elements
  cutoff in $\boldsymbol{y}$ does not break symmetries that haven't already been broken

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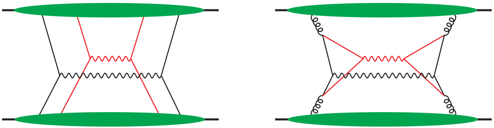

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  analogous regulator for transverse-momentum dependent DPDs
- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order

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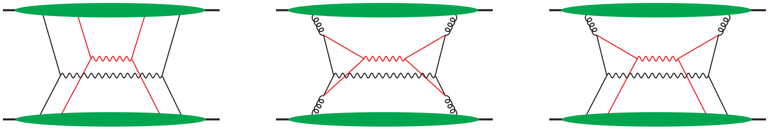
MD, J. Gaunt, K. Schönwald 2017

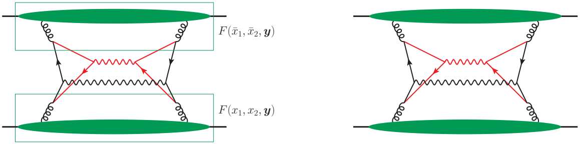


- regulate DPS: $\sigma_{\text{DPS}} \propto \int d^2\boldsymbol{y}\, \Phi^2(\nu y)\, F(x_1, x_2, \boldsymbol{y})\, F(\bar{x}_1, \bar{x}_2, \boldsymbol{y})$
  - $\Phi \to 0$ for $u \to 0$ and $\Phi \to 1$ for $u \to \infty$, e.g. $\Phi(u) = \theta(u-1)$
  - cutoff scale $\nu \sim Q$
  - $F(x_1, x_2, \boldsymbol{y})$ has both splitting and 'intrinsic' contributions

  analogous regulator for transverse-momentum dependent DPDs

- full cross section: $\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub (1v1 + 2v1)}} + \sigma_{\text{SPS}} + \sigma_{\text{tw2}\times\text{tw4}}$
  - subtraction σ_{sub} to avoid double counting:
    $= \sigma_{\text{DPS}}$ with F computed for small $\boldsymbol{y}$ in fixed order perturb. theory
    much simpler computation than σ_{SPS} at given order
  - can also include twist 2 $\times$ twist 4 contribution
    and double counting subtraction for 2v1 term

Subtraction formalism at work

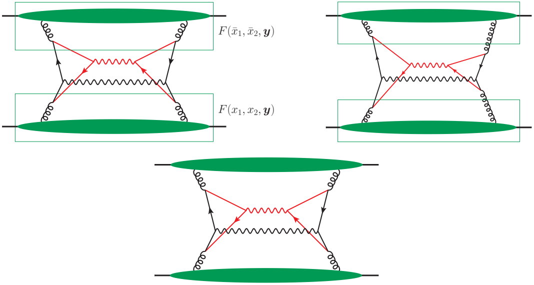


$$\sigma = \sigma_{\text{DPS}} - \sigma_{\text{sub}} + \sigma_{\text{SPS}}$$

- for $y \sim 1/Q$ have $\sigma_{\text{DPS}} \approx \sigma_{\text{sub}}$
  because pert. computation of F gives good approx. at considered order
  $\Rightarrow \quad \sigma \approx \sigma_{\text{SPS}}$ dependence on $\Phi(\nu y)$ cancels between σ_{DPS} and σ_{sub}
- for $y \gg 1/Q$ have $\sigma_{\text{sub}} \approx \sigma_{\text{SPS}}$
  because DPS approximations work well in box graph
  $\Rightarrow \quad \sigma \approx \sigma_{\text{DPS}}$ with regulator fct. $\Phi(\nu y) \approx 1$
- same argument for 2v1 term and $\sigma_{\text{tw2}\times\text{tw4}}$ (were neglected above)
- subtraction formalism works order by order in perturb. theory

J. Collins, Foundations of Perturbative QCD, Chapt. 10

Double counting in TMD factorisation for DPS



- left and right box can independently be collinear or hard:
  $\rightsquigarrow$ DPS, DPS/SPS interference and SPS
- get nested double counting subtractions

M Buffing, MD, T Kasemets 2017

Summary

- multiparton interactions ubiquitous in hadron-hadron collisions
  multiple hard scattering often suppressed, but not necessarily
  - in specific kinematics
  - for multi-differential cross sections, high-multiplicity final states
- double hard scattering depends on detailed hadron structure
  - transverse spatial distribution
  - different correlation and interference effects
- short-distance singularity and double counting with single scattering:
  have consistent solution using cutoff and subtraction terms
- subject of high interest for
  - control over final states at LHC
  - understanding QCD dynamics and hadron structure